Whistler Waves

Whistler is very low frequency or VLF electromagnetic (radio) wave which is generated during lightning discharges or thunderstorms and lightning flash (Fig.1-3). This wave propagates through the ionosphere (the portion of the atmosphere where the number of ions is large enough; it begins at a height of about 50 km above the Earth's surface) which is guided by ducts or region along the earth magnetic field. Frequencies of whistlers are usually much smaller than the electron cyclotron frequency ($\omega << \omega_{ce}$) in the earth ionosphere and is 100 Hz to 10 kHz, with a maximum amplitude usually at 3 kHz to 5 kHz. Yet these waves are electromagnetic waves but they comprise audio frequencies hence can be detected by a sensitive audio amplifier or loudspeaker. Since These waves produces sound thus also called as *whistling atmospheric radio wave*. This wave generates gliding sound or descending pitch whistle from high-to-low-frequency. This is due to that these waves get dispersed in course of time in such a way that the higher frequencies wave move faster than the lower ones. Thus at point of detection, higher frequency wave arrives sooner than the lower ones. When the whistlers are detected at magnetic conjugate points, it is called as *short whistlers*. However, electromagnetic signal may be reflected at the earth surface and get back along the earth magnetic field to a point close to where it is originated. If whistler is detected at this point, it is called as *long whistler(Fig.2)*. Initially, whistlers last about half a second, and they may be repeated at regular intervals of several seconds, growing progressively longer and fainter with time.

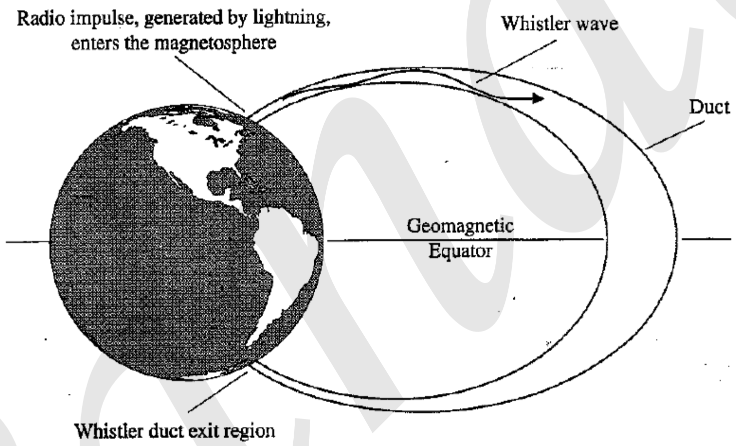


Fig1.

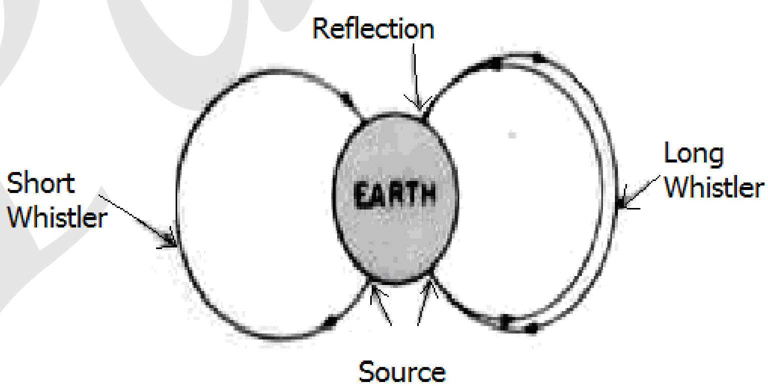


Fig2.

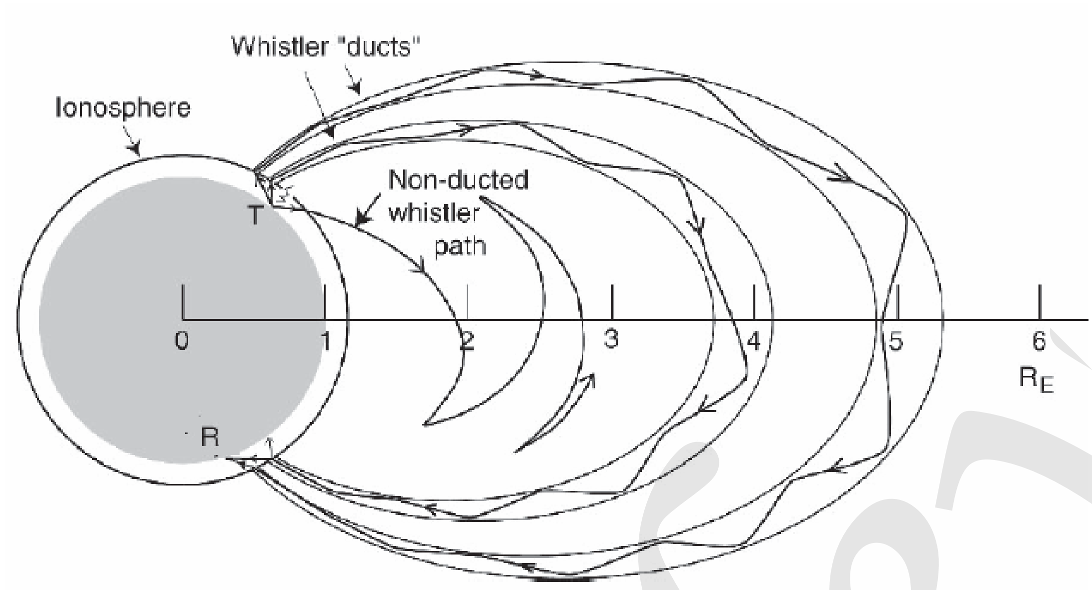


Fig3.

The phenomenon of atmospheric whistler propagation can be explained in terms of very low frequency region of propagation of right circularly polarized wave. Neglecting collision and considering wave propagation nearly along the magnetic field lines under condition $\omega \ll \omega_{ce}$ and $\omega \ll \omega_{pe}$, the Appleton-Hartree equation provides that,

$$\frac{K^2C^2}{\omega^2} = 1 - \frac{X}{1 - Y\cos\theta} \tag{1}$$

Here $X = \frac{\omega_{pe}^2}{\omega^2}$, $Y = \frac{\omega_{ce}}{\omega}$, $\theta =$ angle between wave propagation vector and direction of magnetic field, ω: whistler frequency, ω_{ce} :electron cyclotron frequency and ω_{pe} :electron plasma frequency. The equation (1) is known as dispersion relation for the quasi-longitudinal mode. For $Y\cos\theta \gg 1$, the equation (1) becomes as,

$$\frac{K^2C^2}{\omega^2} = 1 + \frac{X}{Y\cos\theta} \tag{2a}$$

$$\frac{K^2C^2}{\omega^2} = 1 + \frac{\omega_{pe}^2}{\omega\,\omega_{ce}\cos\theta} \tag{2b}$$

If $X \gg Y$ or $\omega_{pe}^2 \gg \omega\,\omega_{ce}$ then equation (2) reduces to,

$$\frac{K^2C^2}{\omega^2} = \frac{\omega_{pe}^2}{\omega\,\omega_{ce}\cos\theta}$$

$$\frac{\omega^2}{K^2} = C^2\,\frac{\omega\,\omega_{ce}\cos\theta}{\omega_{pe}^2}$$

$$\boxed{V_{ph} = \frac{\omega}{K} = C\,\frac{\sqrt{\omega\,\omega_{ce}\cos\theta}}{\omega_{pe}}} \tag{3}$$

Differentiating equation (3) w.r.t. K we have the following expression of group velocity.

$$\boxed{V_g = \frac{\partial \omega}{\partial K} = 2C\frac{\sqrt{\omega\,\omega_{ce}\cos\theta}}{\omega_{pe}}}$$

Thus the group velocity is proportional to the square root of the frequency and consequently the higher frequencies arrive at the receiver slightly ahead of the lower frequencies. That produces the descending pitch when received with simple antenna and loudspeaker. It is found that the maximum angle between the direction of wave propagation of wave packet and magnetic field is 19.5^0. Therefore the wave packet is confined to the beam of less than 200 about the magnetic field lines. At the various locations on the earth, a spectrum of frequency versus time of arrival of these waves is recorded, known as *sonograms of whistler activity* (Fig 4) that is used to study the ionosphere conditions.

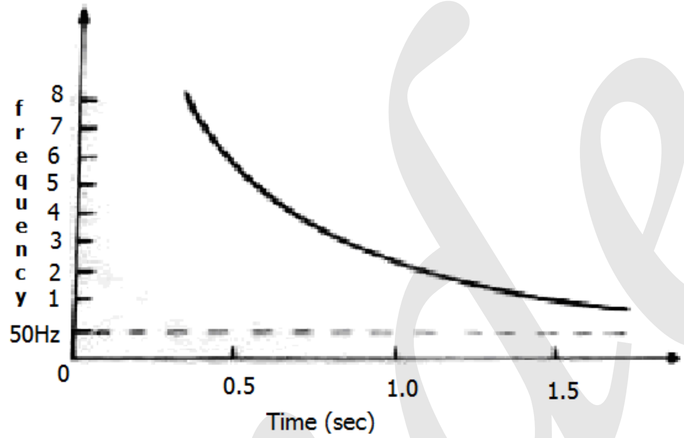


Fig 4

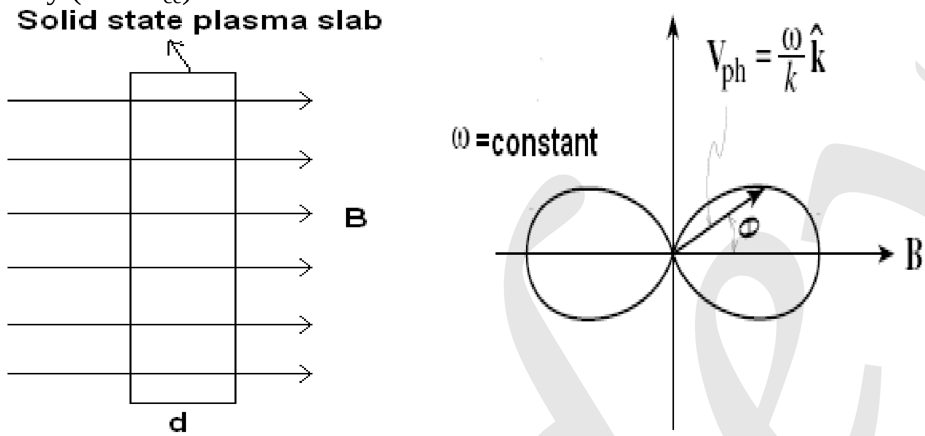
When the frequency of whistler is near (but smaller than) the electron cyclotron frequency, it is possible to observed the frequency increasing with the time of arrival. These are called as ascending frequency whistler. The whistlers in the frequency regime where they change from the ascending to descending tone are known as nose whistler. These types of whistler are also experimentally observed.

Helicon (physics)

A **helicon** is a low frequency electromagnetic wave that can exist in plasmas in the presence of a magnetic field. The *first helicons observed were atmospheric whistlers*, but they also exist in solid conductors or any other electromagnetic plasma. Yet Helicon waves belongs to the category of whistler waves, which are right-hand circularly polarized electromagnetic waves in free space but it differs from classical whistlers in two main respects: (1) they are of such low frequency that the electrons gyrations may be disregarded and only their guiding center motions kept and (2) they are modes of bounded system, in which their purely electromagnetic character cannot be maintained. *Hence helicon wave are produced in solid state plasma which is a very low frequency propagation of right circularly polarized wave. The reason for the name helicon comes from the fact that the tip of wave vector traces a helix.*

Helicons have the special ability to propagate through pure metals at low temperature and high magnetic fields. Most electromagnetic waves in a normal conductor are not able to do this, since the high conductivity of metals acts to screen out the electromagnetic field. Normally an electromagnetic wave would experience a very thin skin depth in a metal. The skin depth is inversely proportionality to the square root of angular frequency. Thus a low frequency electromagnetic wave may be able to overcome the skin depth problem, and thereby propagate throughout the material.

Consider a solid state plasma slab of thickness d whose other two dimensions are very large. Let it is oriented perpendicular to external applied $\vec{B}$ magnetic field as shown in figure (a). Suppose a low frequency right circularly polarized wave is launched in the direction of applied field to slab. The frequency of wave is such that the it is much less than the electron cyclotron frequency ($\omega \ll \omega_{ce}$).



(a) **(b)**

The dispersion relation for the right circularly polarized wave is given by the following equation.

$$\frac{K^2C^2}{\omega^2} = 1 + \frac{\omega_{pe}^2}{\omega\,\omega_{\mathrm{ce}}\cos\theta} \tag{1}$$

Since wave is propagating along $\vec{B}$ thus $\theta = 0$. Alos $\omega_{pe}^2 >> \omega\,\omega_{ce}$, so equation (1) becomes as,

$$\frac{K^2C^2}{\omega^2} = \frac{\omega_{pe}^2}{\omega\,\omega_{\mathrm{ce}}} \qquad \Rightarrow \qquad K^2 = \frac{\omega_{pe}^2}{C^2}\frac{\omega}{\omega_{\mathrm{ce}}}$$

$$K = \frac{\omega_{pe}}{C}\sqrt{\frac{\omega}{\omega_{\mathrm{ce}}}} \tag{2}$$

If K_V is propagation coefficient of the electromagnetic wave in the medium external to the plasma slab then magnitude of reflection coefficient at the plasma boundary is given by,

$$R = \frac{K_V - K}{K_V + K} \approx 1 \qquad \text{as} \qquad \omega \ll \omega_{\mathrm{ce}} \tag{3}$$

Consequently the reflection of wave at the plasma boundary is nearly complete. Therefore the wave reflected at the boundaries of the plasma slab and forms a standing wave whose resonance condition is approximately given by,

$$\frac{n\lambda}{2} = d \qquad \Rightarrow \qquad \frac{n2\pi}{2K} = d \qquad \Rightarrow \qquad \frac{n\pi}{K} = d \tag{4}$$

Here λ is the wavelength of inside the slab of thickness d and n is integer which provides number of standing wave pattern in the slab. From equations (2) and (4), we can write,

$$\frac{n\pi C}{\omega_{pe}}\sqrt{\frac{\omega_{ce}}{\omega}} = d \tag{5a}$$

$$\omega_n = \omega = \left(\frac{n\pi C}{\omega_{pe} d}\right)^2 \omega_{ce} \quad \text{(5b)}$$

Equation (5) is the condition for the standing wave resonance. If frequency of wave launched along $\vec{B}$ is varied continuously such that ω_{pe}, ω_{ce} and d are constant then at frequency $\omega_n = \omega$ there are standing wave resonance inside the plasma slab, resulting a measurable large wave amplitude. If parameters d, ω_{pe} and ω are kept constant and $\vec{B}$ is varied then the standing wave resonance is obtained at,

$$\omega_{ce} = (\omega_{pe})_n = \left(\frac{\omega_{pe} d}{n\pi C}\right)^2 \omega \quad \text{(6)}$$

Thus we can say that when low frequency electromagnetic wave travels in solid state plasma slab along the applied magnetic field then due reflection at plasma boundary and standing wave resonance, the helicon wave is formed.

<u>Faraday rotation</u>

In 1845, Michael Faraday discovered the first physical phenomenon linking light and magnetism. He was able to rotate the polarization of light when he induced a magnetic field in the same direction as the path of the light. This effect, known as the Faraday rotation or Faraday Effect, only occurs when light passes through transparent dielectrics.

When a plane polarized electromagnetic wave (light wave) is sent through plasma or certain materials along parallel magnetic field then the plane of polarization of wave get rotated. This phenomenon of rotation of plane of polarization of wave is called as Faraday rotation.

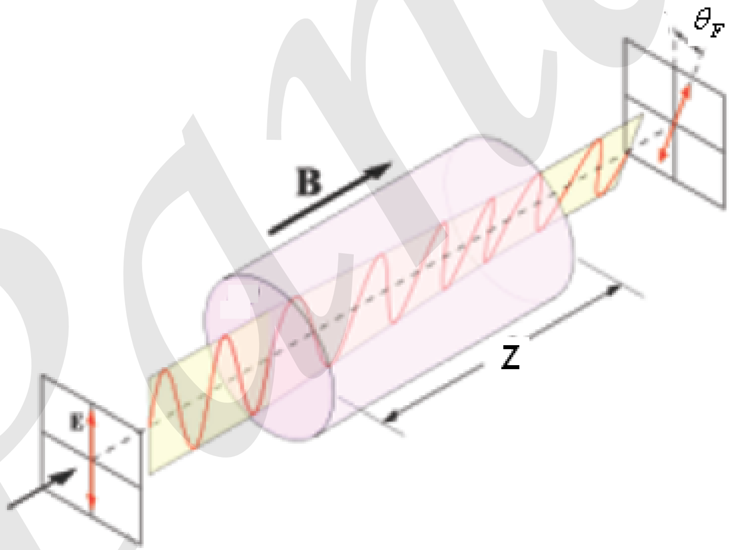


Fig.1

A plane polarized wave is considered as superposition of Left and right circularly polarized waves which propagate independently (Fig.2). The phenomenon of Faraday rotation is due to difference in phase velocity of both circularly polarized waves in presence of external field. The RCP wave propagates faster than the LCP waves. Suppose, after travelling a certain distance, the faster RCP wave completes *n* cycles while slower LCP wave completes *n*+ *ε* *(ε>0)*. Since both waves are considered to have same frequency thus resultant of them or plane of polarization is rotated counterclockwise direction (Fig3).

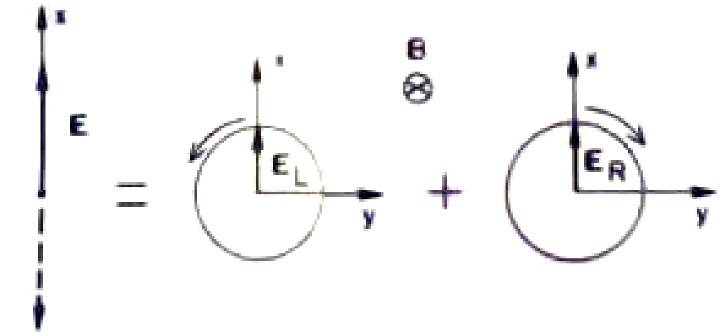


Fig.2

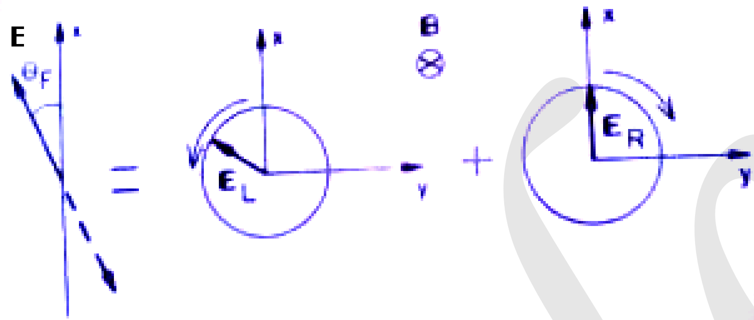


Fig.3

Let a plane polarized wave propagates in plasma in z-direction along the direction of external applied magnetic field. At z=0, the electric field component has only the x-component (fig 2). Thus the electric field component at z=0 can be written as,

$$\vec{E}(z=0,t) = E_0 \exp(\text{-i}\omega\text{t})\,\hat{x} \tag{1}$$

This equation can be re-written as,

$$\vec{E}(z=0,t) = \left[\frac{E_0}{2}\ (\hat{x}+i\hat{y}) + \frac{E_0}{2}\ (\hat{x}-i\hat{y})\right]\exp(\text{-i}\omega\text{t}) \tag{2}$$

Here the first and second terms in the right hand side of equation (2) are RCP and LCP components. These two components propagate independently so that for any z>0, the electric field vector can be given as,

$$\vec{E}(z,t) = \left[\frac{E_0}{2}\ (\hat{x}+i\hat{y})\exp(\text{ik}_\text{R} z) + \frac{E_0}{2}\ (\hat{x}-i\hat{y})\exp(\text{ik}_\text{L} z)\right]\exp(\text{-i}\omega\text{t}) \tag{3}$$

Here, k_R and k_L are the wave vector for the RCP and LCP waves. Re-arranging equation (3) we have,

$$\vec{E}(z,t) = \frac{E_0}{2}\exp\left(\frac{\text{i(k}_\text{R}+k_L)z}{2}\right)\left[\ (\hat{x}+i\hat{y})\exp\left(\frac{\text{i(k}_\text{R}-k_L)z}{2}\right) + \ (\hat{x}-i\hat{y})\exp\left(\frac{\text{-i(k}_\text{R}-k_L)z}{2}\right)\right]\exp(\text{-i}\omega\text{t})$$

$$\vec{E}(z,t) = \frac{E_0}{2}\exp\left(\frac{\text{i(k}_\text{R}+k_L)z}{2}\right)\left[\ \hat{x}\cos\left(\frac{(\text{k}_\text{R}-k_L)z}{2}\right) - \hat{y}\sin\left(\frac{(\text{k}_\text{R}-k_L)z}{2}\right)\right]\exp(\text{-i}\omega\text{t}) \tag{4}$$

Equation (4) represents a linearly polarized wave at z whose plane of polarization is rotated in counterclockwise direction by the angle θ_F such that,

$$\boxed{\theta_F = \frac{(\text{k}_\text{R}-k_L)z}{2}} \quad \text{or} \quad \frac{\theta_F}{z} = \frac{(\text{k}_\text{R}-k_L)=z}{2} \tag{5}$$

Therefore the angle of rotation per unit distance (θ_F/z) depends on the difference between the propagation coefficients of RCP and LCP waves. The measurement of Faraday rotation is useful tool in plasma diagnostic and is widely used in the investigation of ionosphere properties.