Reflection, Refraction and Polarization of EM wave

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Boundary condition for EM wave

1. The normal component of electric displacement vector is not continuous at the interface but changes by an amount equal to the free surface charge density.

$$D_{1n} - D_{2n} = \sigma$$

2. The normal component of magnetic induction B is continuous across the interface.

$$B_{1n} - B_{2n} = 0$$

Boundary condition for EM wave

3. The tangential component of electric field is continuous at the interface.

$$E_{1t} - E_{2t} = 0$$

4. The tangential component of magnetic field strength is not continuous at the interface but changes by an amount equal to the component of the surface current density perpendicular to tangential component of H.

$$H_{1t} - H_{2t} = J_{S\perp}$$

Reflection and Refraction of EM wave

For Incident wave

$$\vec{E}_1 = \vec{E}_{01} e^{i(\vec{K}_1 . \vec{r} - \omega t)}$$

$$\vec{B}_1 = \frac{\vec{K}_1 \times \vec{E}_1}{\omega_1}$$

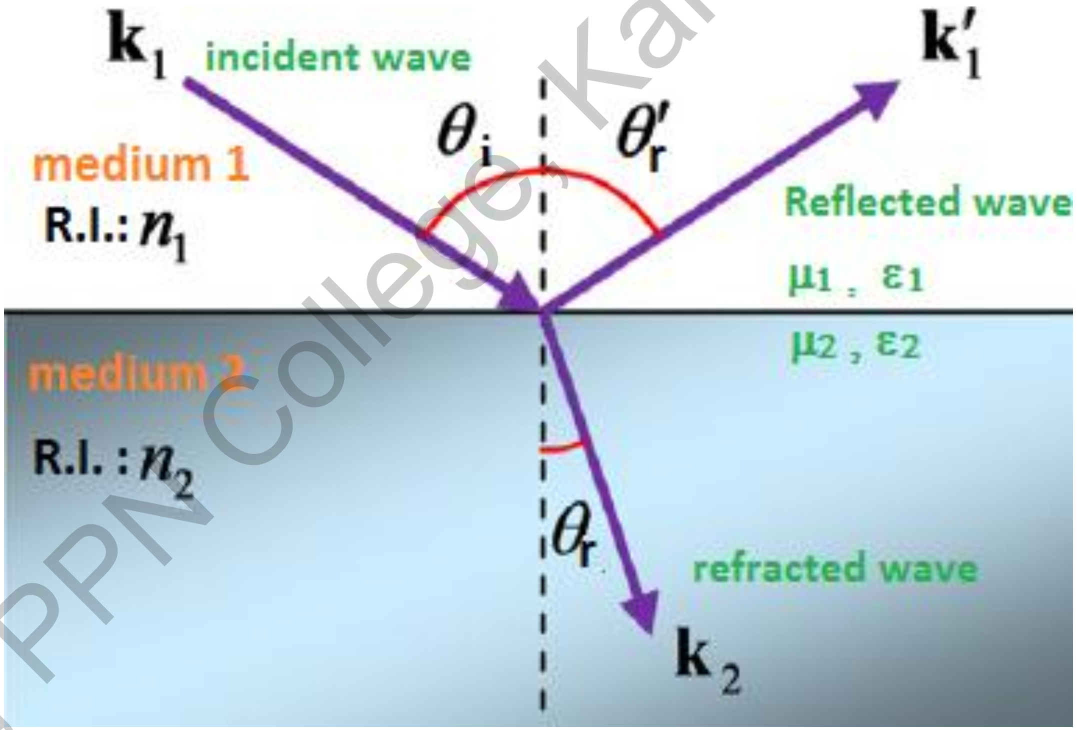
$$\vec{H}_1 = \frac{\vec{K}_1 \times \vec{E}_1}{\mu_1 \omega_1}$$

For reflected wave

$$\vec{E}'_1 = \vec{E}'_{01} e^{i(\vec{K}'_1 . \vec{r} - \omega t)}$$

$$\vec{B}'_1 = \frac{\vec{K}'_1 \times \vec{E}'_1}{\omega'_1}$$

$$\vec{H}'_1 = \frac{\vec{K}'_1 \times \vec{E}'_1}{\mu_1 \omega'_1}$$



For refracted wave

$$\vec{E}_2 = \vec{E}_{02} e^{i(\vec{K}_2 . \vec{r} - \omega t)}$$

$$\vec{B}_2 = \frac{\vec{K}_2 \times \vec{E}_2}{\omega_2} \qquad \vec{H}_1 = \frac{\vec{K}_1 \times \vec{E}_1}{\mu_1 \omega_1}$$

Reflection and Refraction of EM wave

Since, tangential component of electric field is continuous at interface .

$$(E_1)_t + (E'_1)_t = (E_2)_t$$

$$(E_{01})_t e^{i(\vec{k}_1.\vec{r}-\omega_1 t)} + (E'_{o1})_t e^{i(\vec{k}'_1.\vec{r}-\omega'_1 t)} = (E_{02})_t e^{i(\vec{k}_2.\vec{r}-\omega_2 t)}$$

$$\because \omega_1 = \omega'_1 = \omega_2 = \omega$$

$$\therefore (E_{01})_t e^{i(\vec{k}_1.\vec{r})} + (E'_{o1})_t e^{i(\vec{k}'_1.\vec{r})} = (E_{02})_t e^{i(\vec{k}_2.\vec{r})}$$

$$\text{Therefore, for} \quad (E_{01})_t + (E'_{o1})_t = (E_{02})_t$$

$$\therefore \quad (\vec{k}_1.\vec{r})_{z=0} = (\vec{k}'_1.\vec{r})_{z=0} = (\vec{k}_2.\vec{r})_{z=0}$$

Reflection and Refraction of EM wave

$$\text{If}, \quad (\vec{k}_1.\vec{r})_{z=0} = (\vec{k}'_1.\vec{r})_{z=0}$$

$$k_1 x \sin\theta_i = k'_1 x \sin\theta'_r$$

Since wave propagation vector does not vary in same medium thus

$$\therefore k_1 = k'_1$$

$$\sin\theta_i = \sin\theta'_r$$

$$\theta_i = \theta'_r$$

Law of reflection

$$\text{If}, \quad (\vec{k}_1.\vec{r})_{z=0} = (\vec{k}_2.\vec{r})_{z=0}$$

$$k_1 x \sin\theta_i = k_2 x \sin\theta_r$$

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{k_2}{k_1}$$

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{k_2}{\omega}\frac{\omega}{k_1} = \frac{v_1}{v_2}$$

$$n = \frac{\sin\theta_i}{\sin\theta_r} = \frac{v_1}{v_2} = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}$$

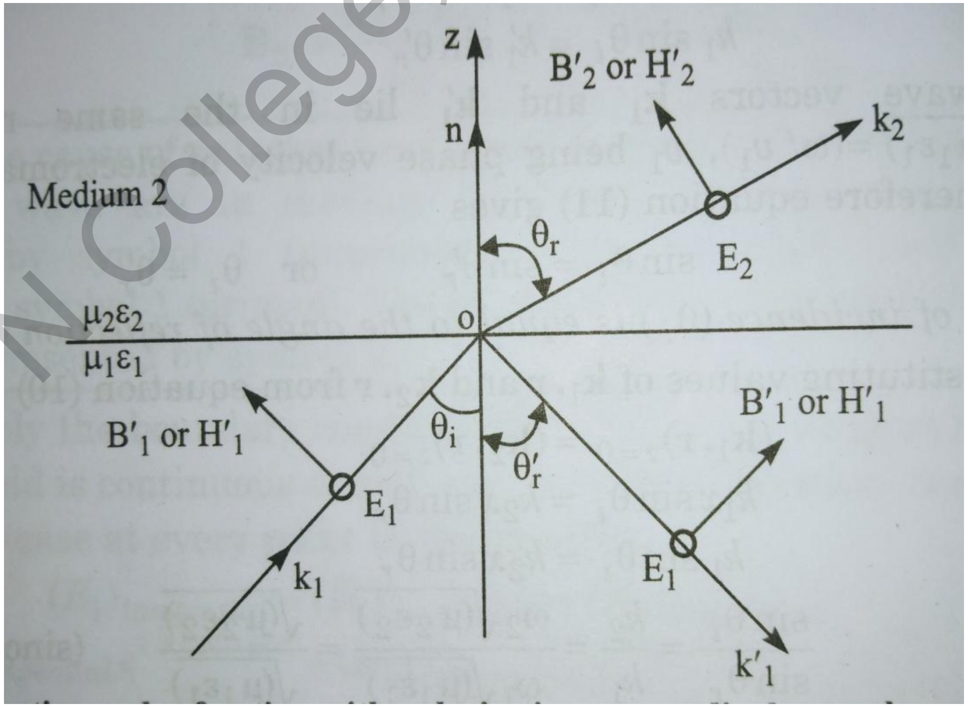
Law of refraction

Fresnel Equation
for Reflection and Refraction of EM wave

A: When E is perpendicular to plane of incidence

$$\frac{E'_{01}}{E_{01}} = \frac{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_r}{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_r}$$

$$\frac{E_{02}}{E_{01}} = \frac{2\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i}{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_r}$$



Fresnel Equation………… contd for Reflection and Refraction of EM wave

$$\text{if} \quad \mu_1 = \mu_2 = \mu_0 \quad \text{then} \; \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\sin\theta_i}{\sin\theta_r}$$

$$\frac{E'_{01}}{E_{01}} = \frac{n_1\cos\theta_i - n_2\cos\theta_r}{n_1\cos\theta_i + n_2\cos\theta_r} = \frac{\sin(\theta_i - \theta_r)}{\sin(\theta_i + \theta_r)}$$

$$\frac{E_{02}}{E_{01}} = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_r} = \frac{2\cos\theta_i\sin\theta_r}{\sin(\theta_i + \theta_r)}$$

For normal incidence ; $\theta_i = \theta_r = 0$

$$\frac{E'_{01}}{E_{01}} = \frac{n_1 - n_2}{n_1 + n_2} \qquad \frac{E_{02}}{E_{01}} = \frac{2n_1}{n_1 + n_2}$$

Fresnel Equation………… contd for Reflection and Refraction of EM wave

Case 1: when $(n_1/n_2)<1$; wave : rare to denser

$n = \sin\theta_i / \sin\theta_r = (n_2/n_1) > 1$ and $\theta_i > \theta_r$

$$\frac{E'_{01}}{E_{01}} = -ve \Rightarrow \text{Phase Change} = \pi \ ; \text{Path difference} = \frac{\lambda}{2}$$

$$\frac{E_{02}}{E_{01}} = +ve \Rightarrow \text{Phase Change} = 0$$

Reflected and incident wave are in opposite phase.

Refracted and incident wave are in same phase.

Case 2: when $(n_1/n_2)>1$; wave :denser to rare

$n = \sin\theta_i / \sin\theta_r = (n_2/n_1) < 1$ and $\theta_i < \theta_r$

$$\frac{E'_{01}}{E_{01}} = +ve \Rightarrow \text{Phase Change} = 0$$

$$\frac{E_{02}}{E_{01}} = +ve \Rightarrow \text{Phase Change} = 0$$

Reflected and incident wave are in same phase.

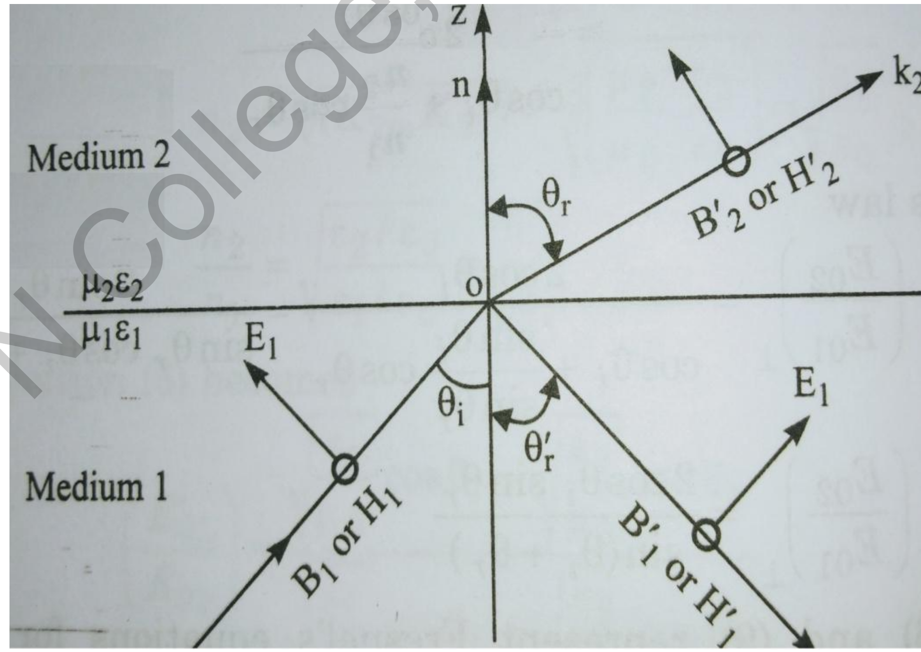
Refracted and incident wave are in same phase.

Fresnel Equation………… contd
for Reflection and Refraction of EM wave

B: When E is parallel to plane of incidence

$$\frac{E'_{01}}{E_{01}} = \frac{\sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_i - \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_r}{\sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_i + \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_r}$$

$$\frac{E_{02}}{E_{01}} = \frac{2\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i}{\sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_i + \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_r}$$



Fresnel Equation………… contd for Reflection and Refraction of EM wave

$$\text{if} \quad \mu_1 = \mu_2 = \mu_0 \quad \text{then} \; \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\sin\theta_i}{\sin\theta_r}$$

$$\frac{E'_{01}}{E_{01}} = \frac{n_2\cos\theta_i - n_1\cos\theta_r}{n_2\cos\theta_i + n_1\cos\theta_r} = \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)}$$

$$\frac{E_{02}}{E_{01}} = \frac{2n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_r} = \frac{2\cos\theta_i\sin\theta_r}{\sin(\theta_i + \theta_r)\cos(\theta_i - \theta_r)}$$

For normal incidence ; $\theta_i = \theta_r = 0$

$$\frac{E'_{01}}{E_{01}} = \frac{n_2 - n_1}{n_2 + n_1} \qquad \frac{E_{02}}{E_{01}} = \frac{2n_1}{n_2 + n_1}$$

Fresnel Equation………… Contd….
for Reflection and Refraction of EM wave

Case 1: $$\frac{E_{02}}{E_{01}} = +ve \Rightarrow \text{Phase Change} = 0$$

Refracted and incident wave are in same phase.

Case 2: From both cases we found that -

$$r_{II} = \left(\frac{E'_{01}}{E_{01}}\right)_{II} = \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)}; \quad r_{\perp} = \left(\frac{E'_{01}}{E_{01}}\right)_{\perp} = \frac{\sin(\theta_i - \theta_r)}{\sin(\theta_i + \theta_r)}$$

$$\text{if} \quad \theta_i + \theta_r = \frac{\pi}{2} \text{ ,or} \quad \theta_p = \theta_B = \theta_i = \frac{\pi}{2} - \theta_r = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

$$r_{II} = 0 \quad \text{and} \quad r_{\perp} \neq 0$$

Thus , if an un-polarized light incidents (rare to denser) at angle **$\theta p = \theta_B$** then only electric vector perpendicular to plane of incidence will be reflected and light will become polarized. This angle is termed as angle of polarization or Brewster's angle.

Fresnel Equation………… Contd….
for Reflection and Refraction of EM wave

Case 3: if em wave travels from denser to rarer medium then it goes away from normal.

$$\theta_r = \frac{\pi}{2} \quad \text{or} \quad \theta_i = \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$r_{II} = \left(\frac{E'_{01}}{E_{01}}\right)_{II} = \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)}; \quad r_{\perp} = \left(\frac{E'_{01}}{E_{01}}\right)_{\perp} = \frac{\sin(\theta_i - \theta_r)}{\sin(\theta_i + \theta_r)}$$

$$r_{II} = 1 \text{ and } r_{\perp} = 1$$

$$R_{II} = (r_{II})^2 = 1 \text{ and } R_{\perp} = (r_{\perp})^2 = 1$$

Thus total energy is reflected at the interface of two mediums.

This means, as the angle of incident wave increases, the intensity of reflected wave increases while intensity of refracted wave diminishes. The intensity of refracted wave becomes zero at $\theta i = \theta c$. But if $\theta i > \theta c$ then wave completely goes in medium Ist, and follows law of reflection. This is TIR.

Reflection and transmission coefficient

$$R = \frac{\text{Intensity of reflected wave}}{\text{Intensity of incident wave}} = \frac{v_1(E'_{01})^2}{v_1(E_{01})^2} = \frac{(E'_{01})^2}{(E_{01})^2}$$

$$T = \frac{\text{Intensity of transmitted wave}}{\text{Intensity of incident wave}} = \frac{v_2(E_{02})^2}{v_1(E_{01})^2} = \frac{k_1(E_{02})^2}{k_2(E_{01})^2}$$

$$T = \frac{n_2(E_{02})^2}{n_1(E_{01})^2}$$

For normal incidence ; $\theta_i = \theta_r = 0$

$$\frac{E'_{01}}{E_{01}} = \frac{n_2 - n_1}{n_2 + n_1} \qquad \frac{E_{02}}{E_{01}} = \frac{2n_1}{n_2 + n_1} \quad \Rightarrow \quad R + T = 1$$

Note : Degree of polarization

$$P = \frac{R_\perp - R_{II}}{R_\perp + R_{II}} \; ; \; R_\perp = \frac{\sin^2(\theta_i - \theta_r)}{\sin^2(\theta_i + \theta_r)} \; \& \; R_{II} = \frac{\tan^2(\theta_i - \theta_r)}{\tan^2(\theta_i + \theta_r)}$$

Faraday Effect or Faraday Rotation

When an isotropic medium of high refractive index is placed in strong magnetic field, then it becomes optically active temporarily.

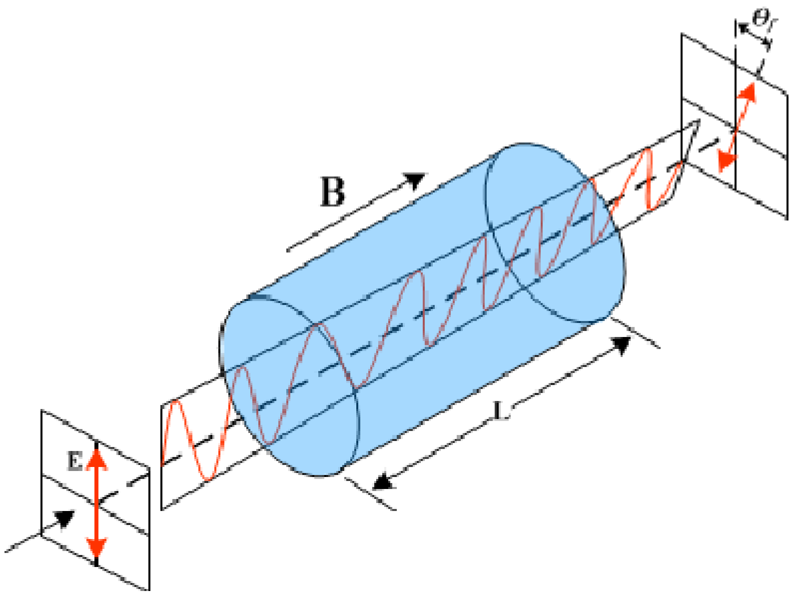
If a plane polarized EM wave passes through such medium then its plane of polarization is rotated. This is called as Faraday rotation.

$$\theta \propto BL$$

$$\theta = VBL$$

V=Verdet constant

$$\text{If}, B = 1T \text{ and } L = 1m$$
$$V = \theta$$



A Lot of Thanks

for attention