Network Theorems

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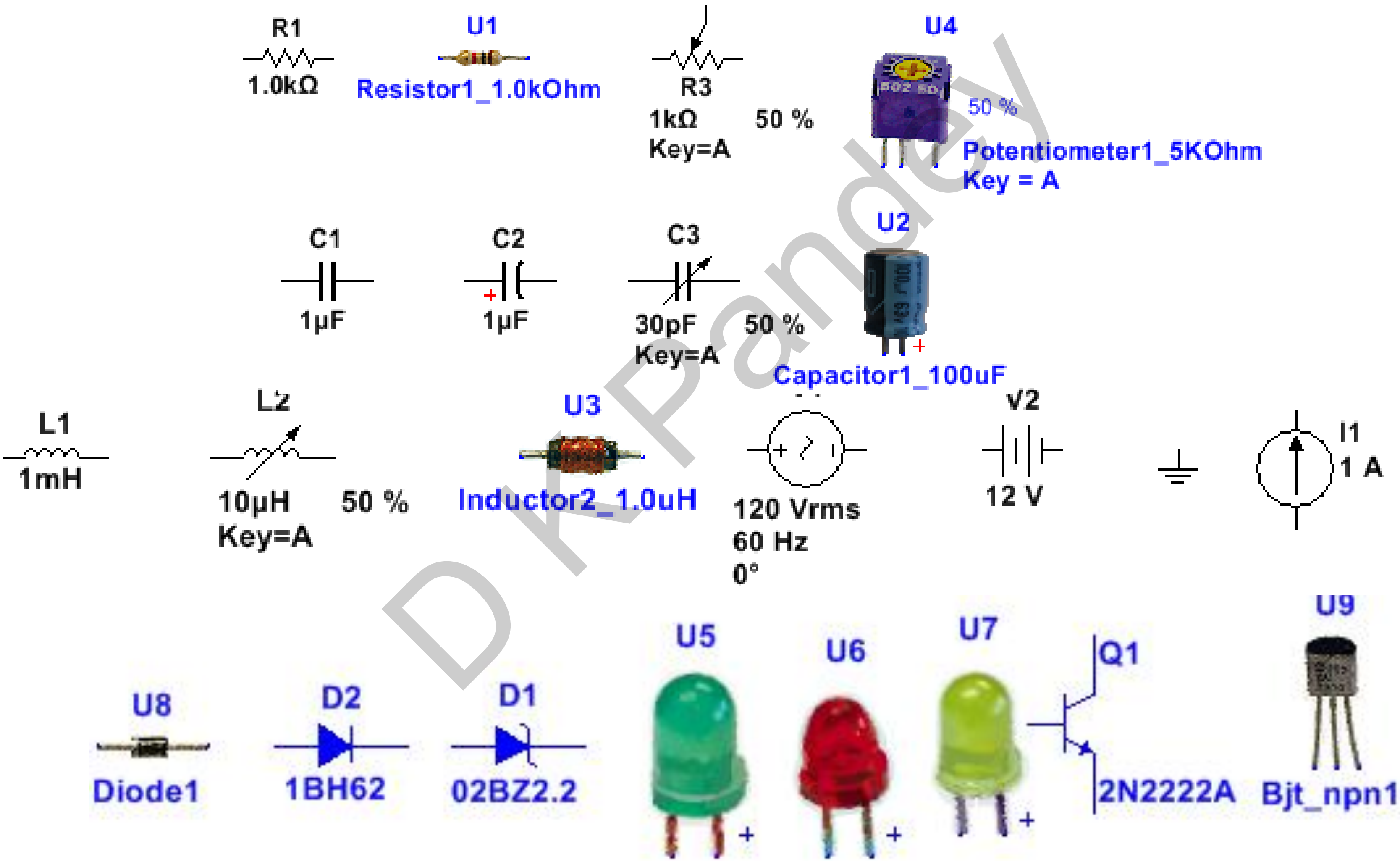
Components


R1
1.0kΩ
U1
Resistor1_1.0kOhm
R3
1kΩ
50 %
Key=A
U4
50 %
Potentiometer1_5KOhm
Key = A
C1
1µF
C2
1µF
C3
30pF
50 %
Key=A
U2
Capacitor1_100uF
L1
1mH
L2
10µH
50 %
Key=A
U3
Inductor2_1.0uH
120 Vrms
60 Hz
0°
V2
12 V
I1
1 A
U8
Diode1
D2
1BH62
D1
02BZ2.2
U5
U6
U7
Q1
2N2222A
U9
Bjt_npn1

Active and Passive components

Passive component:

The electrical component through which energy is dissipated are called as passive component. e.g. resistor, inductor, and capacitor

Active component:

The circuit component which generates energy is called as active component. e.g. dc or ac sources etc.

Electrical Network

Electrical network/circuit:

The interconnection of electrical circuit components (resistors, capacitors, inductors and energy sources) which results a closed path is called as electrical network.

Active network:

An electrical circuit containing both the active and passive components is called as active network.

Passive network:

An electrical circuit containing only passive components is called as passive network.

Linear and non-linear Electrical Network

Linear network:

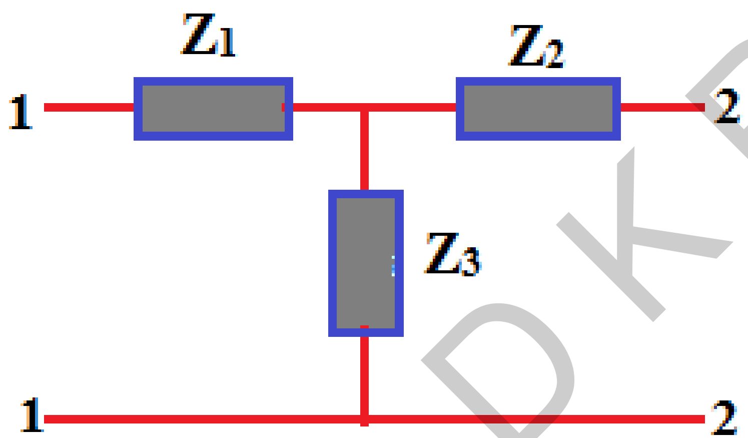
If current in electrical circuit is directly proportional to the source voltage then the network is termed as linear network. i.e. there is linear relationship between current and voltage for this network.

Non-linear network:

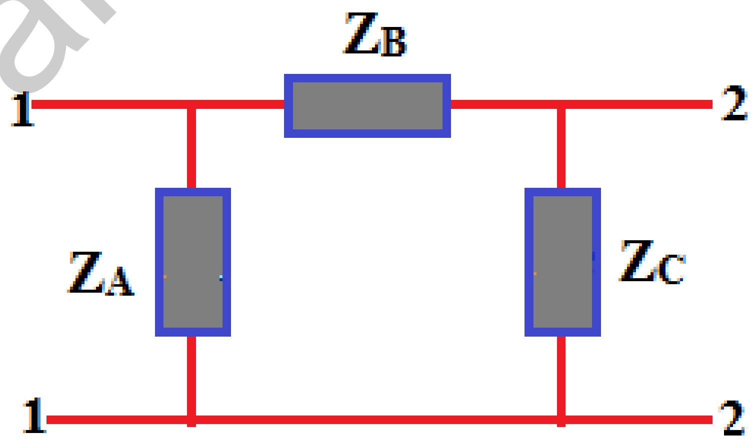
If current in electrical circuit is not directly proportional to the source voltage then the network is termed as non-linear network. i.e. there is non-linear relationship between current and voltage for this network.

Four terminal electrical Network

Four terminal network: *If an electrical network has two input and two output terminal then it is called as four terminal network. e.g. T-network or Y-network and π-network or ∇-network.*

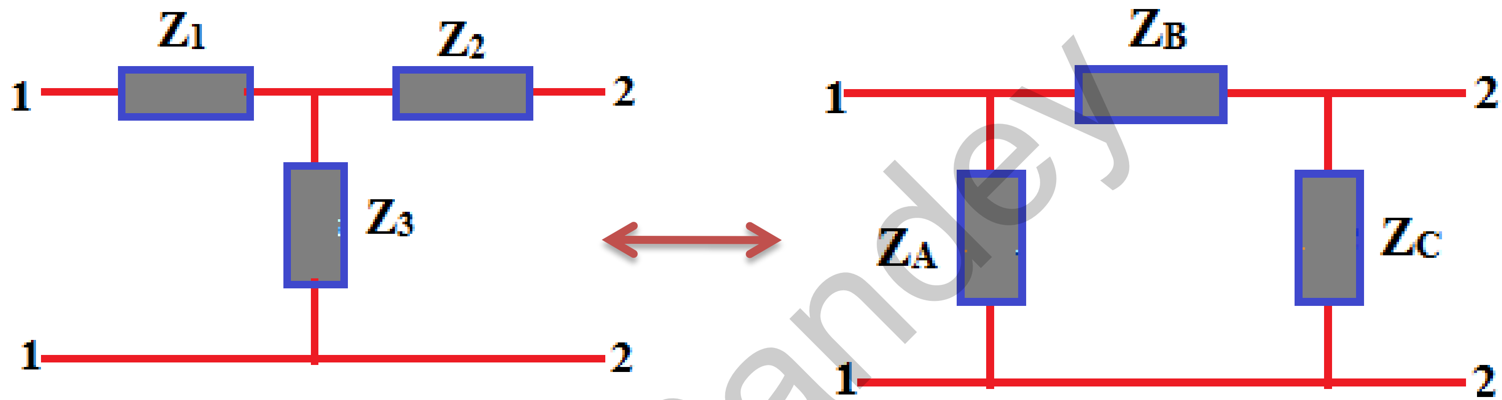


T or Y network



π or ∇ network

Conversion of T $\leftrightarrow \pi$ Network



$$Z_A = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

$$Z_C = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

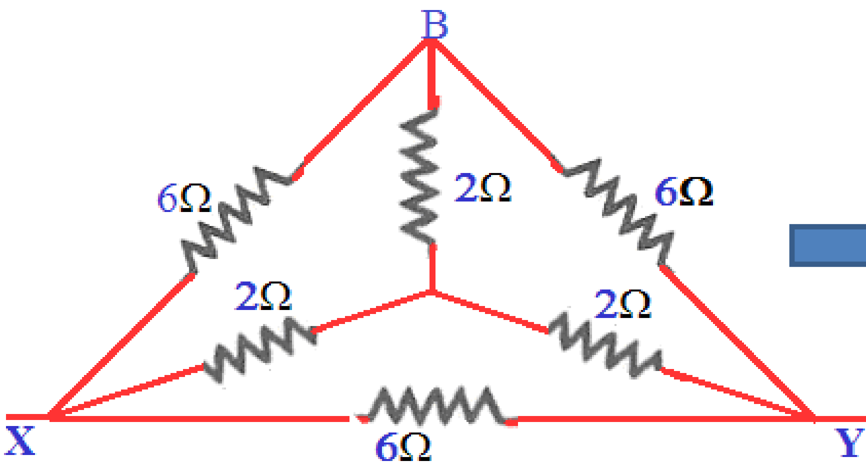
$$\longleftrightarrow$$

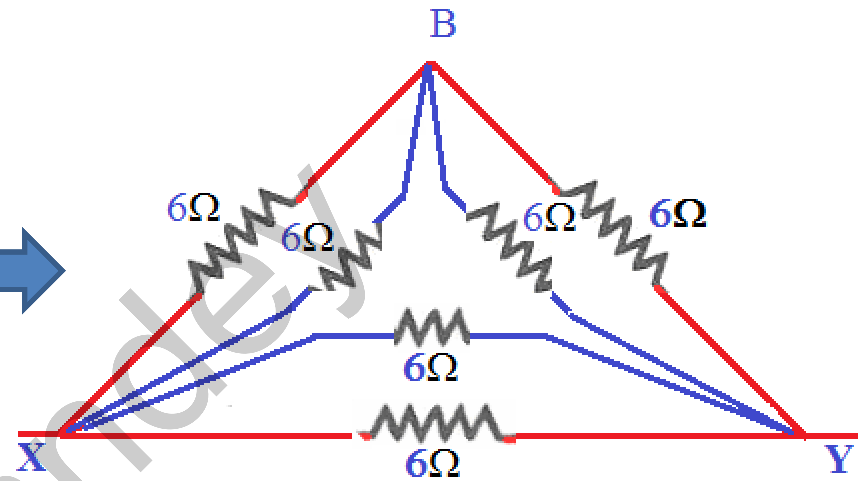
$$Z_1 = \frac{Z_AZ_B}{Z_A + Z_B + Z_C}$$

$$Z_2 = \frac{Z_BZ_C}{Z_A + Z_B + Z_C}$$

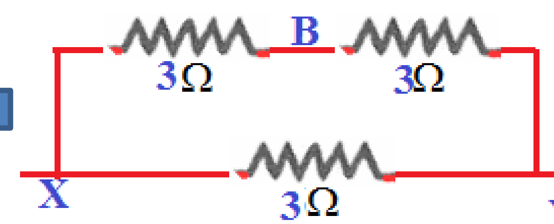
$$Z_3 = \frac{Z_CZ_A}{Z_A + Z_B + Z_C}$$

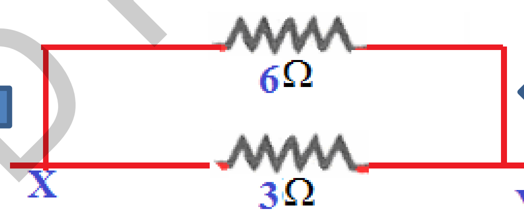
Example





$$Z_A = Z_B = Z_C = \frac{2\times 2 + 2\times 2 + 2\times 2}{2} = \frac{12}{2} = 6\Omega$$

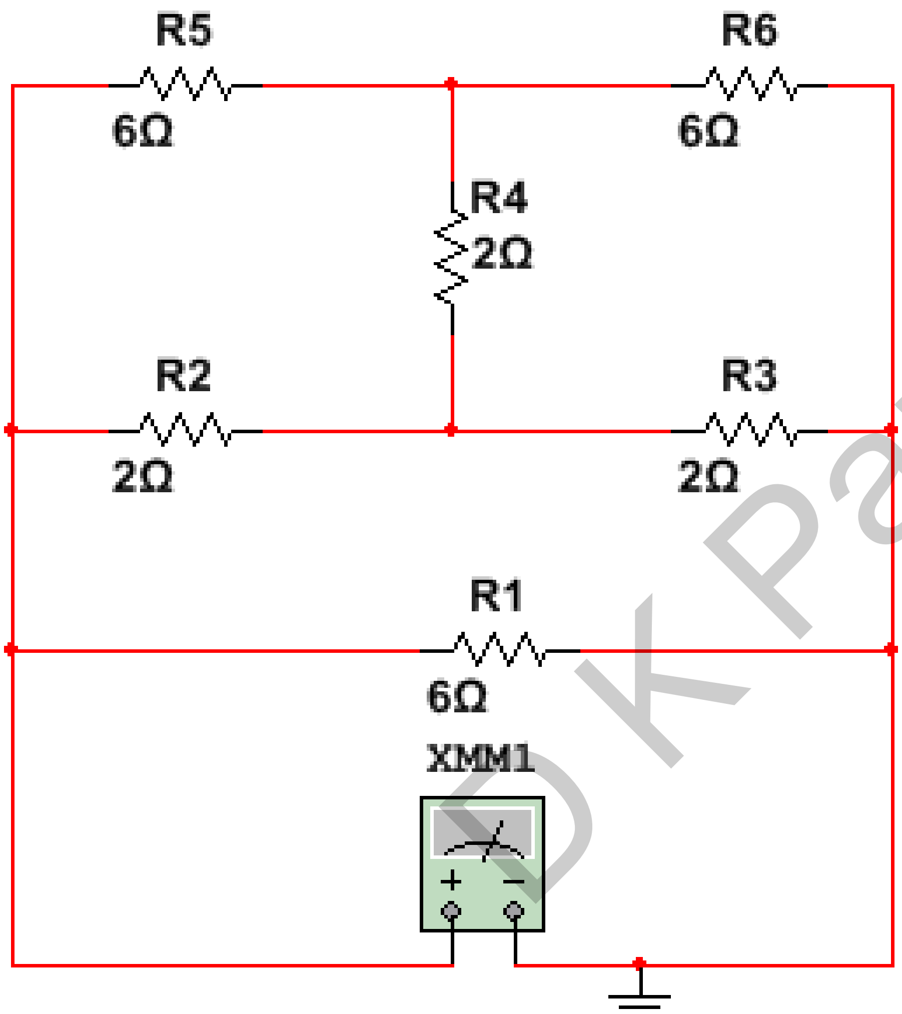


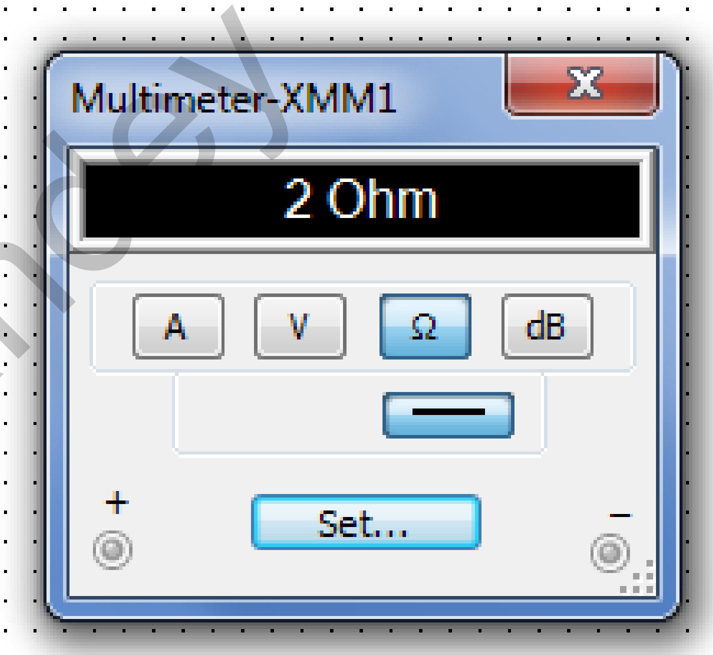


$$Z_{XY} = R_{XY} = \frac{6\times 3}{6+3} = \frac{18}{9} = 2\Omega$$

$$Z_{XB} = Z_{BY} = \frac{6\times 6}{6+6} = \frac{36}{12} = 3\Omega$$

Out put of same circuit by software





Basic Law of electrical network

Ohm's Law: $$\boxed{V = iR}$$

Kirchoff's Current Law (KCL): *The algebraic sum of currents at node in electrical network is equal to zero.*

Or

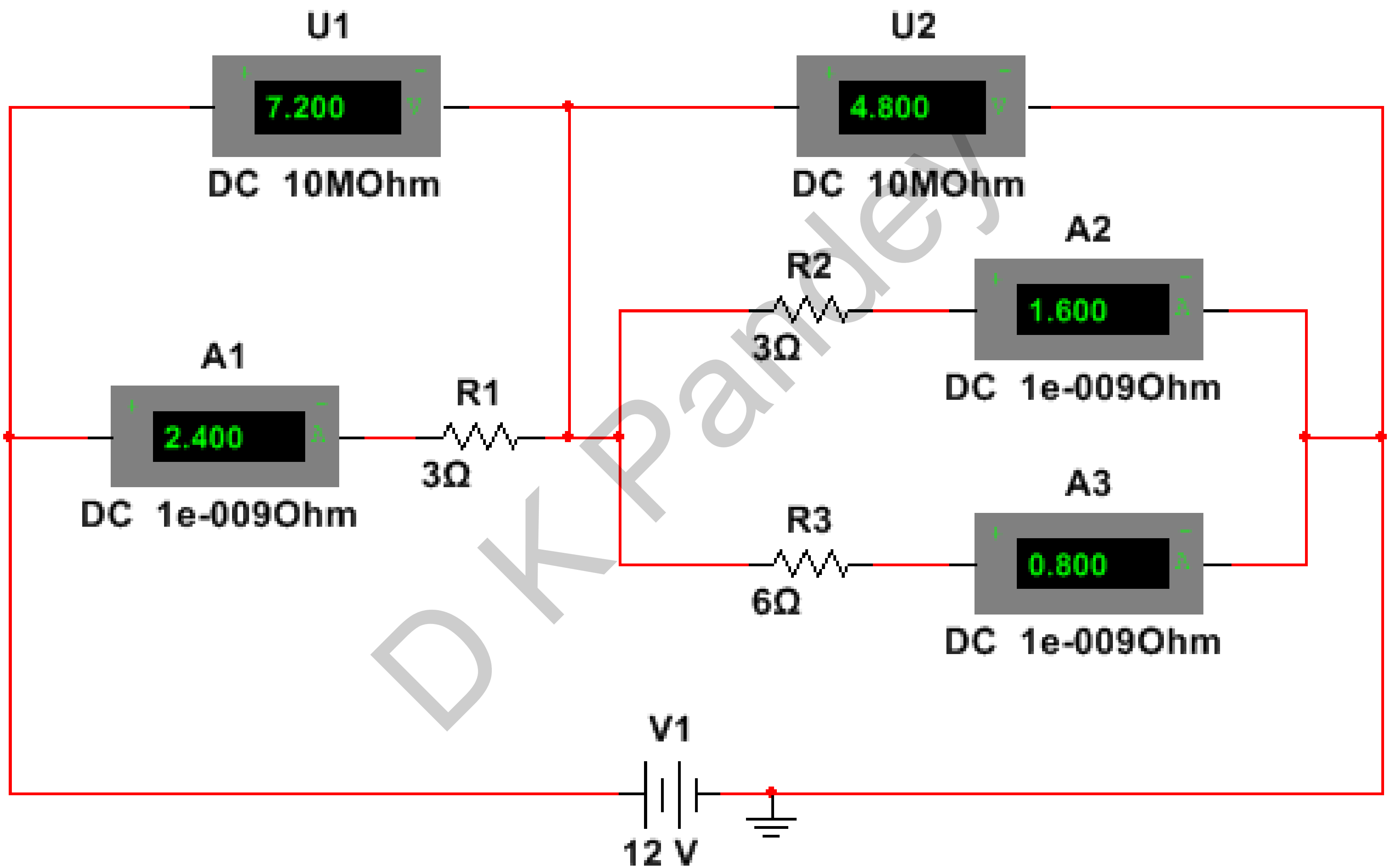
$$\boxed{\sum i = 0} \Rightarrow \boxed{\begin{array}{l} \text{Incoming current} - \text{Out going current} = 0 \\ \text{Incoming current} = \text{Out going current} \end{array}}$$

Kirchoff's Vltage law (KVL): *The algebraic sum of instantaneous voltage drop across the circuit elements for a closed loop is zero.*

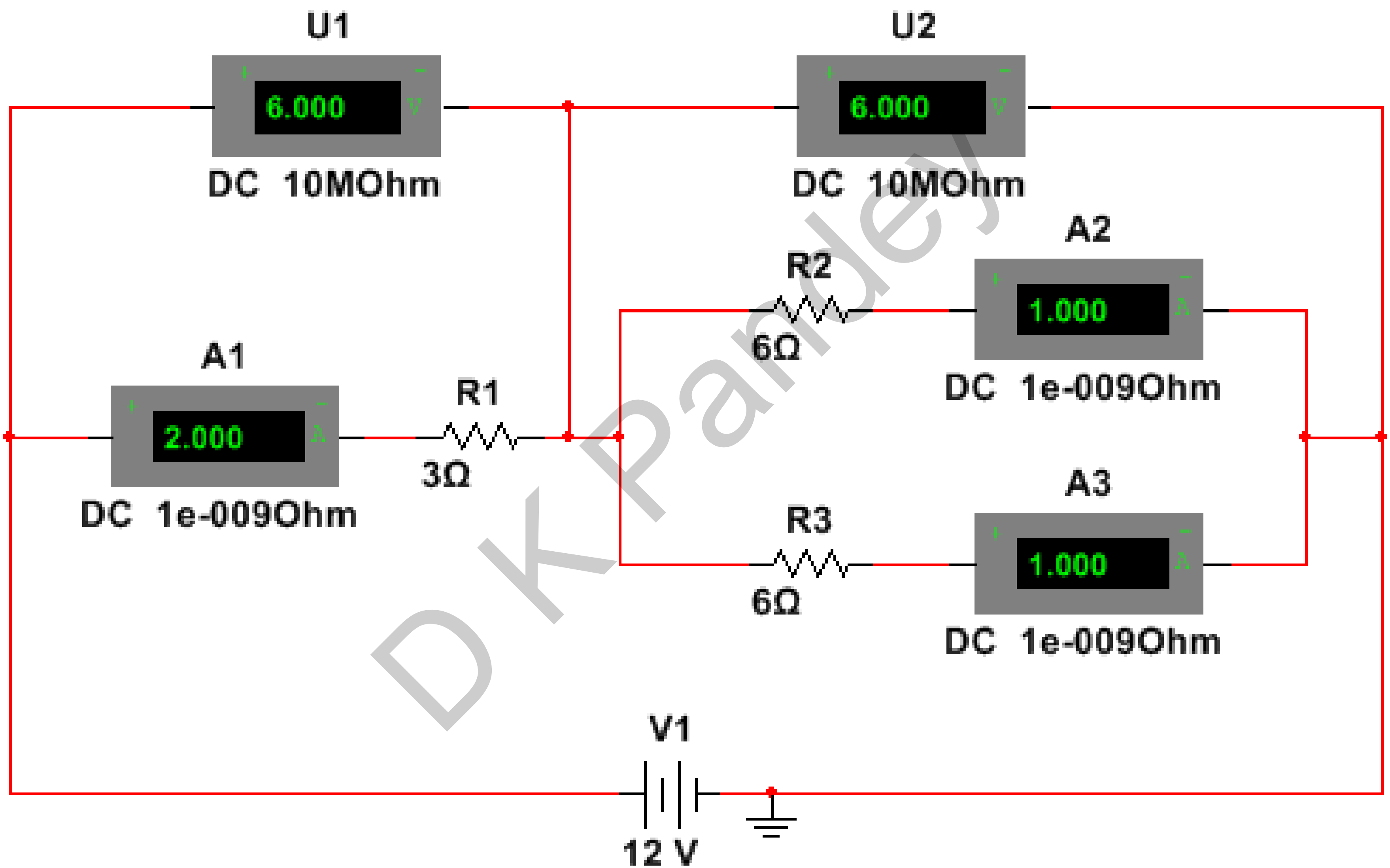
$$\sum V = 0 \Rightarrow \sum iR - \sum E = 0 \Rightarrow \sum iR = \sum E$$

$$\boxed{\sum iZ = \sum E}$$

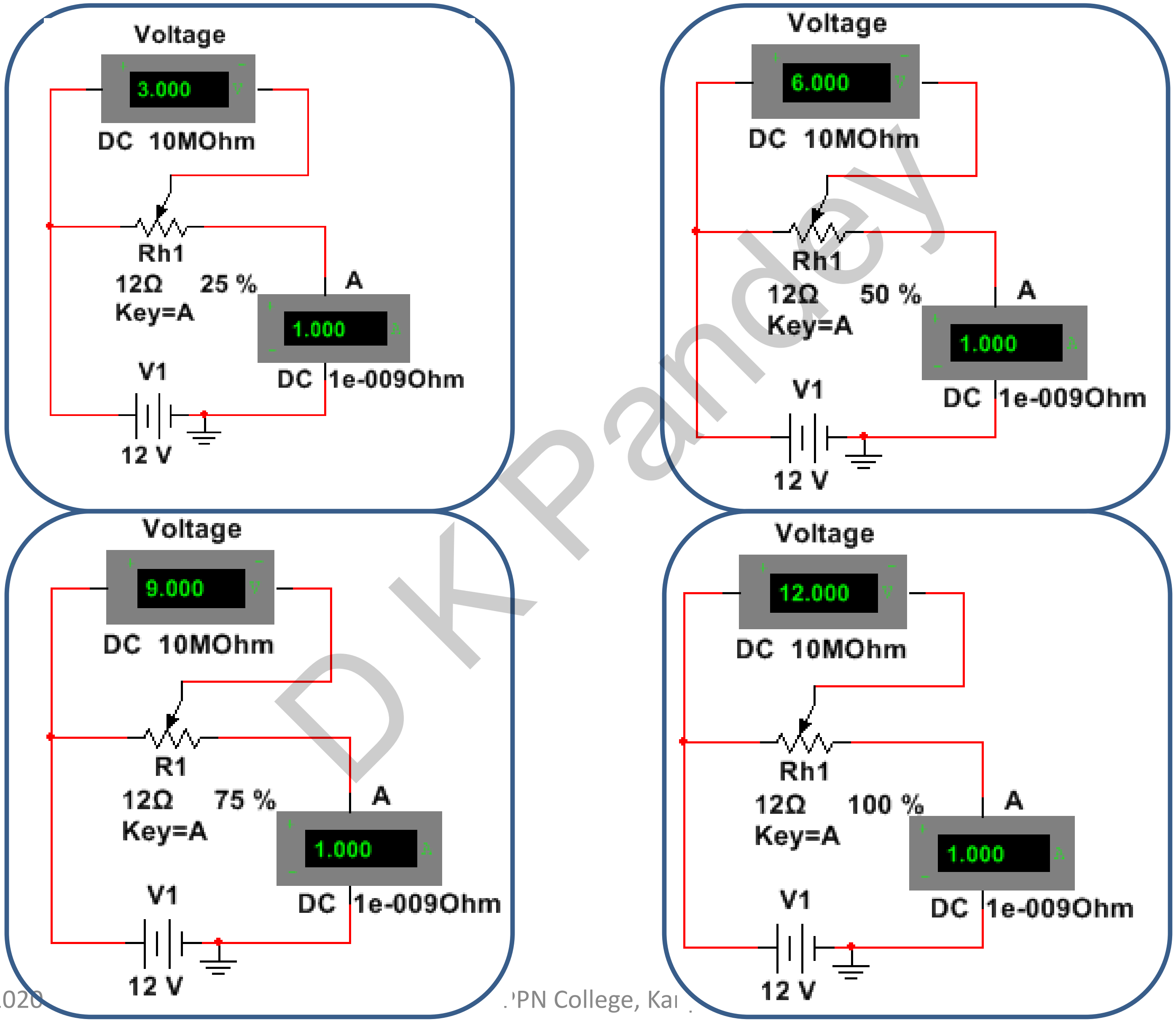
Example for Basic Laws



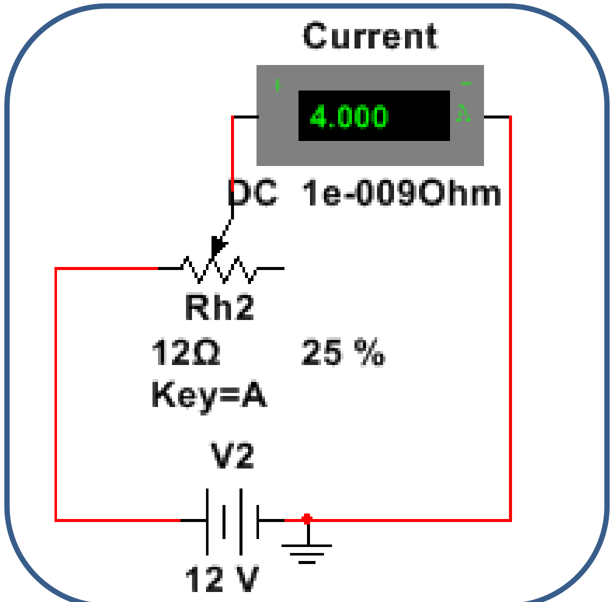
Example for Basic Laws

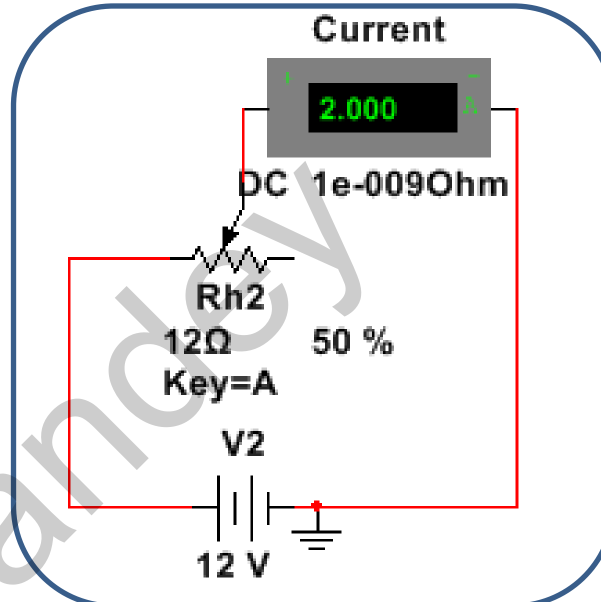


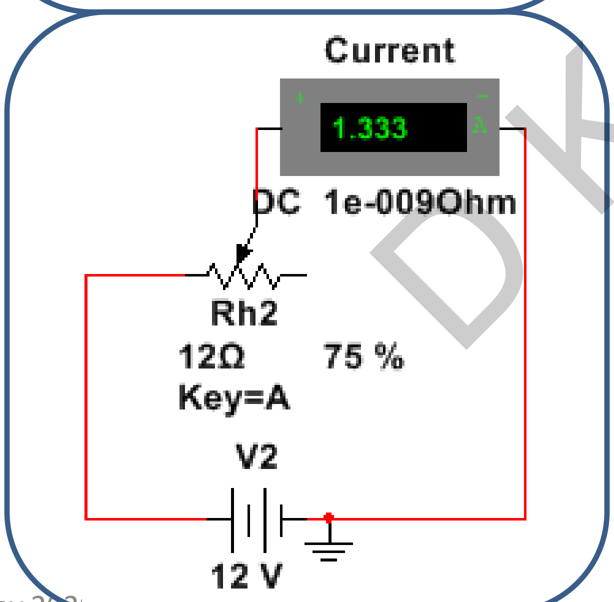
Example for Basic Laws- Potential divider

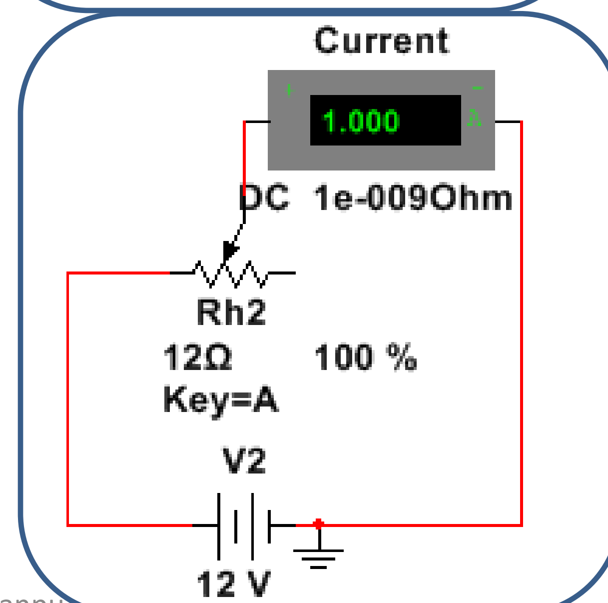


Example for Basic Laws – Current control







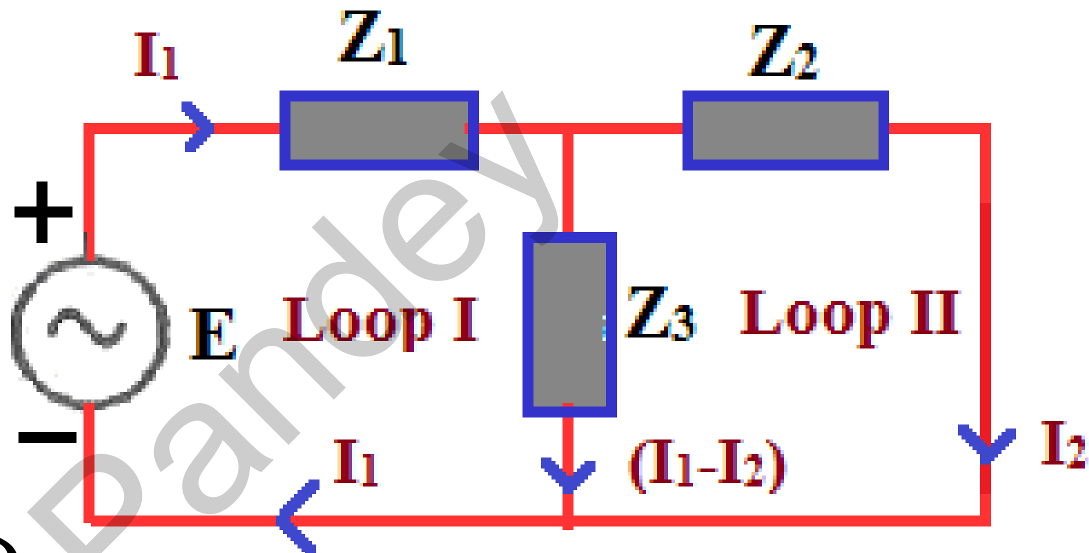


Application of Kirchhoff's Law for two loop network

Applying KVL for loop I,

$$Z_1 I_1 + Z_3(I_1 - I_2) = E$$

$$(Z_1 + Z_3)I_1 - Z_3 I_2 = E \quad (1)$$



Applying KVL for loop II,

$$\mathbf{Z_2 I_2 - Z_3(I_1 - I_2) = 0}$$

$$\mathbf{-Z_3 I_1 + (Z_2 + Z_3)I_2 = 0 \quad (2)}$$

Solving equations (1) and (2),

$$I_1 = \frac{E(Z_2 + Z_3)}{(Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1)}$$

$$I_2 = \frac{Z_3 E}{(Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1)}$$

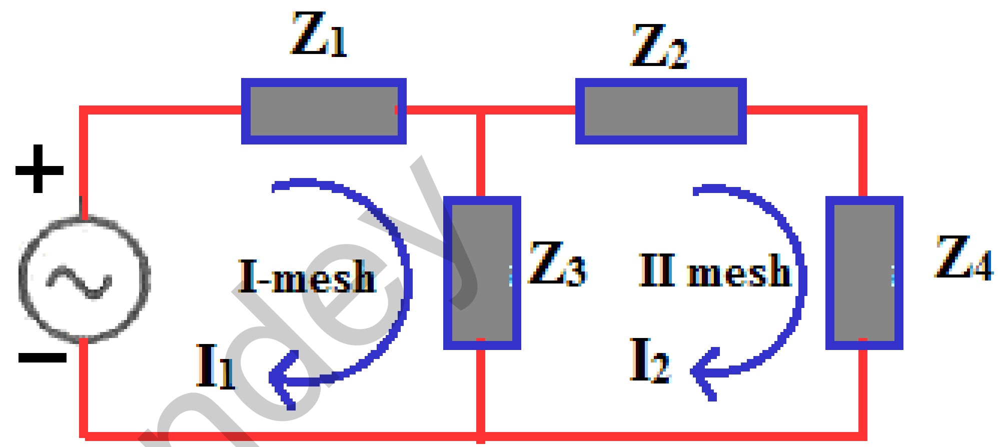
Mesh analysis of two loop Network

Mesh equations for loop I

$$Z_{11} I_1 + Z_{12} I_2 = E_1$$

Mesh equations for loop I

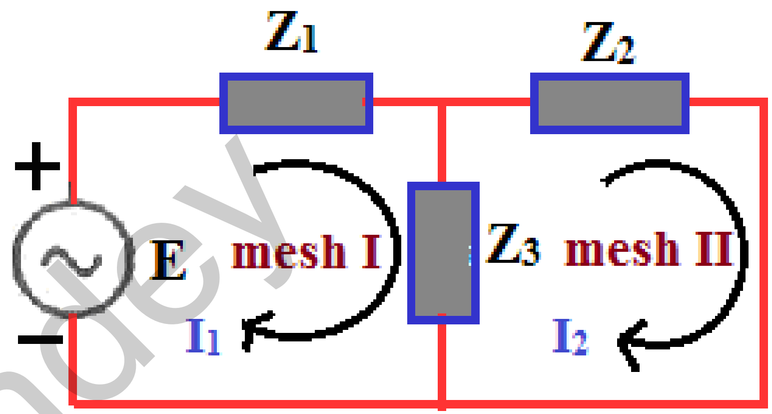
$$Z_{21} I_1 + Z_{22} I_2 = E_2$$



Here E_1 and E_2 are algebraic sum of e.m.f. of energy sources in mesh 1 and 2 respectively. **Z_{11} and Z_{22}** are called as **loop impedance** of mesh first and second respectively. **Z_{12} and Z_{21}** are called **as mutual impedance** of between meshes 1 and 2.

$$I_1 = \frac{\begin{bmatrix} E_1 & Z_{12} \\ E_2 & Z_{22} \end{bmatrix}}{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}} \qquad I_2 = \frac{\begin{bmatrix} Z_{11} & E_1 \\ Z_{21} & E_2 \end{bmatrix}}{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}}$$

Application of Mesh analysis for two loop Network



The mesh equation for mesh I and mesh II can be written as,

$$Z_{11}I_1 + Z_{12}I_2 = E_1$$

$$Z_{21}I_1 + Z_{22}I_2 = E_2$$

Here,

$$Z_{11} = Z_1 + Z_3 \qquad Z_{12} = -Z_3 \qquad E_1 = E$$

$$Z_{21} = -Z_3 \qquad Z_{22} = Z_2 + Z_3 \qquad E_2 = 0$$

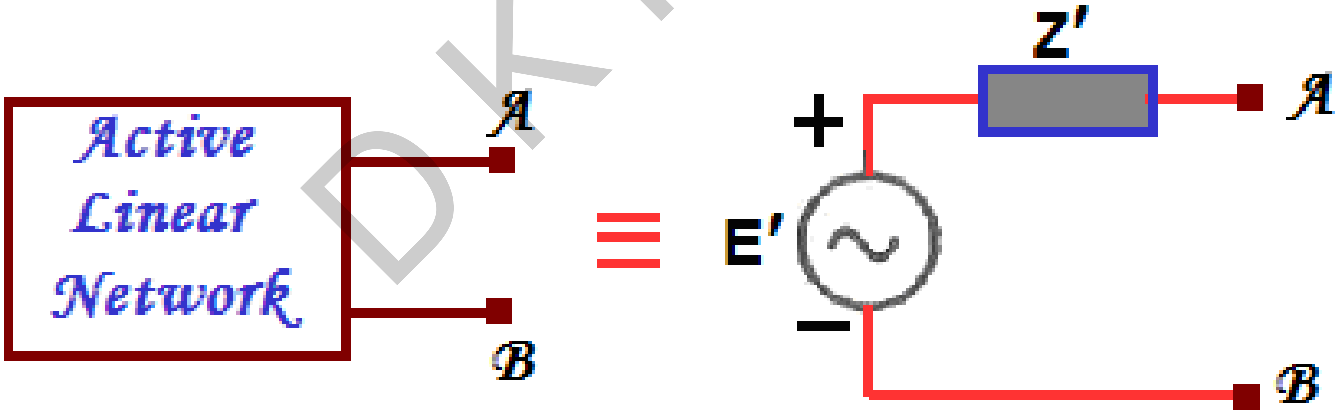
So,

$$I_1 = \frac{\begin{bmatrix} E_1 & Z_{12} \\ E_2 & Z_{22} \end{bmatrix}}{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}} = \frac{\begin{bmatrix} E & -Z_3 \\ 0 & Z_2 + Z_3 \end{bmatrix}}{\begin{bmatrix} Z_1 + Z_3 & -Z_3 \\ -Z_3 & Z_2 + Z_3 \end{bmatrix}} \Rightarrow \boxed{I_1 = \frac{E(Z_2 + Z_3)}{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1)}}$$

$$I_2 = \frac{\begin{bmatrix} Z_{11} & E_1 \\ Z_{21} & E_2 \end{bmatrix}}{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}} = \frac{\begin{bmatrix} Z_1 + Z_3 & E \\ -Z_3 & 0 \end{bmatrix}}{\begin{bmatrix} Z_1 + Z_3 & -Z_3 \\ -Z_3 & Z_2 + Z_3 \end{bmatrix}} \Rightarrow \boxed{I_2 = \frac{Z_3E}{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1)}}$$

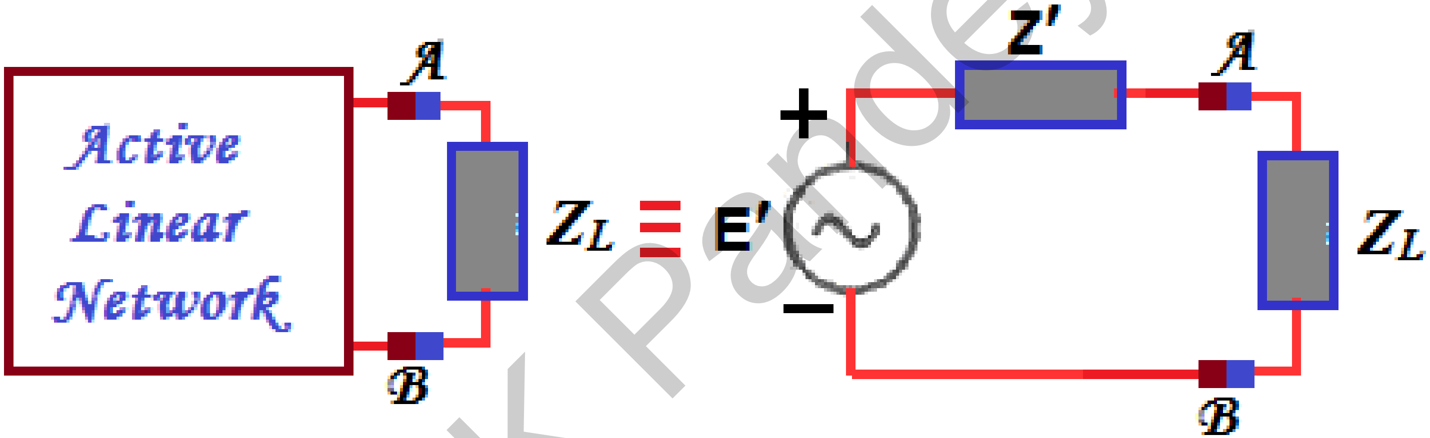
Thevenin's Theorem

Statement: *The theorem states that "Any two terminal linear network containing energy sources and impedances (active network) is equivalent to a voltage source of E´ in combination with impedance Z´ in series. Where E´* is open circuited voltage across the terminals of network and *Z´* is the impedance of network when the sources are replaced by their internal impedances or short circuited.

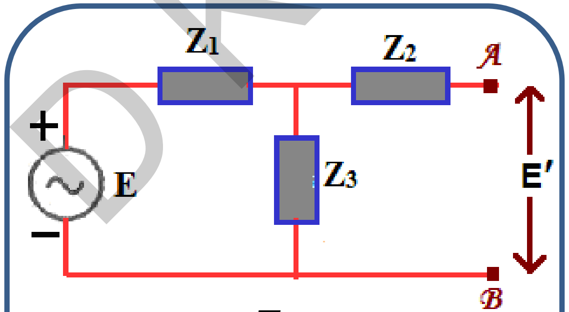


Thevenin's Theorem ……..continued

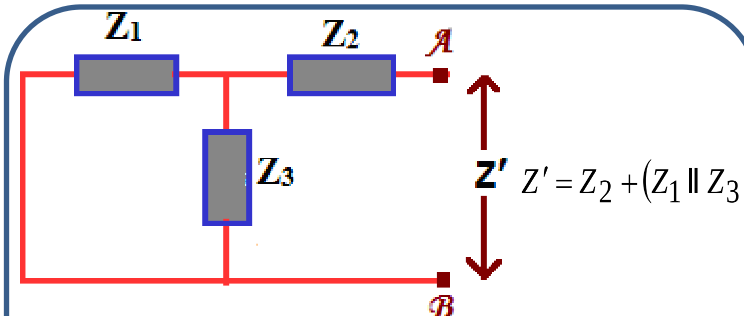
If a load impedance Z_L is connected to the terminal A and B of the network then load current I_L can be written as,



$$I_L = \frac{E'}{Z' + Z_L}$$



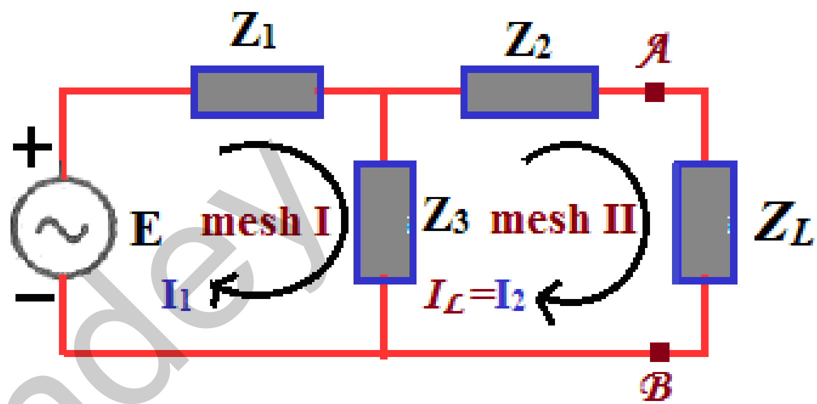
$$E' = \frac{Z_3}{(Z_1 + Z_3)} E$$



$$Z' = Z_2 + \frac{Z_1 Z_3}{(Z_1 + Z_3)}$$

Proof of Thevenin's Theorem

The mesh equation for mesh I and mesh II can be written as,



$$\boxed{\begin{aligned} Z_{11}I_1 + Z_{12}I_2 &= E_1 \\ Z_{21}I_1 + Z_{22}I_2 &= E_2 \end{aligned}}$$

Here,

$$Z_{11} = Z_1 + Z_3 \qquad Z_{12} = -Z_3 \qquad E_1 = E$$

$$Z_{21} = -Z_3 \qquad Z_{22} = Z_2 + Z_3 + Z_L \qquad E_2 = 0$$

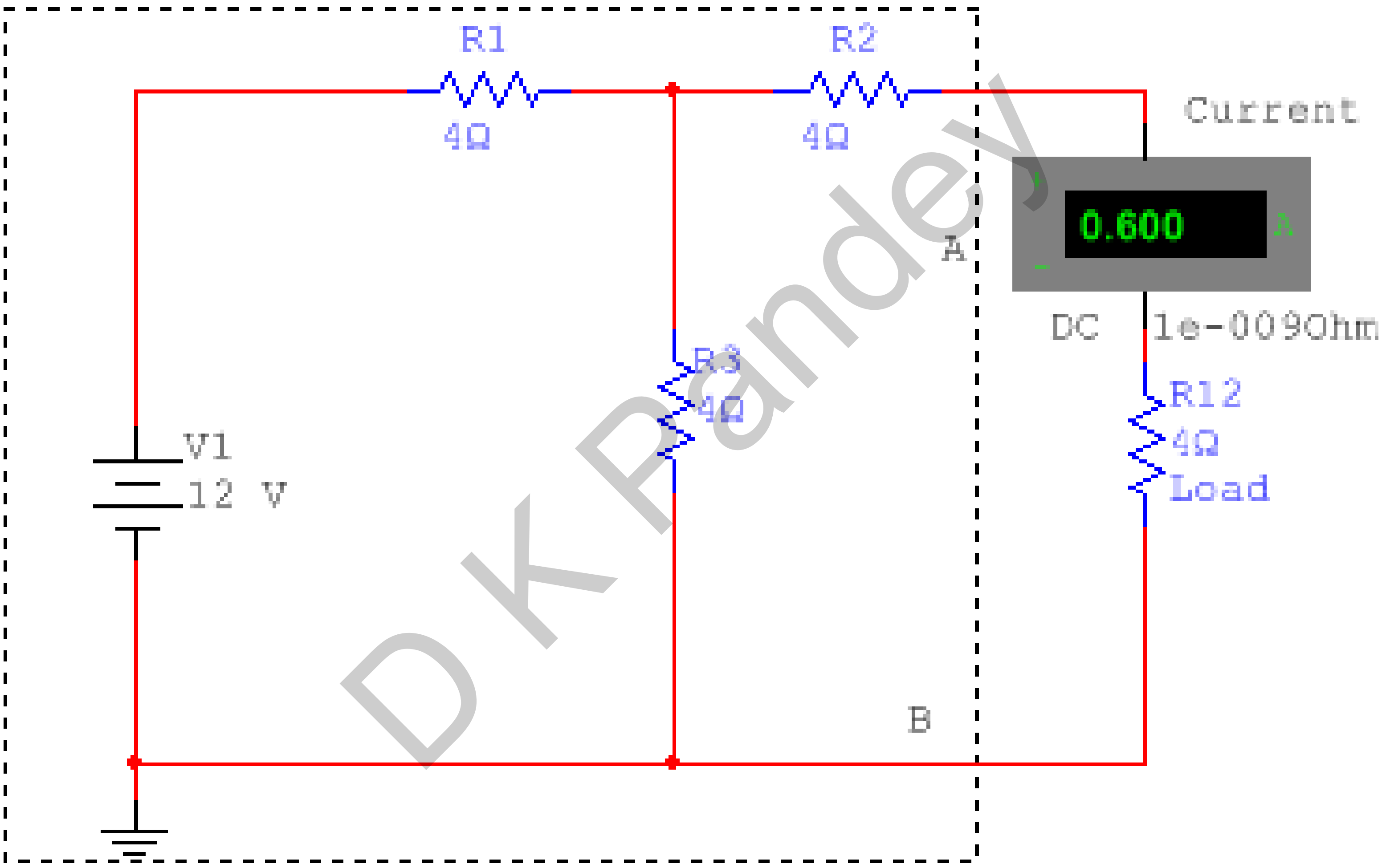
So,

$$I_2 = \frac{\begin{bmatrix} Z_{11} & E_1 \\ Z_{21} & E_2 \end{bmatrix}}{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}} = \frac{\begin{bmatrix} Z_1 + Z_3 & E \\ -Z_3 & 0 \end{bmatrix}}{\begin{bmatrix} Z_1 + Z_3 & -Z_3 \\ -Z_3 & Z_2 + Z_3 + Z_L \end{bmatrix}} \Rightarrow I_2 = I_L = \frac{0 - (-Z_3)E}{\{(Z_1 + Z_3)(Z_2 + Z_3 + Z_L) - Z_3^2\}}$$

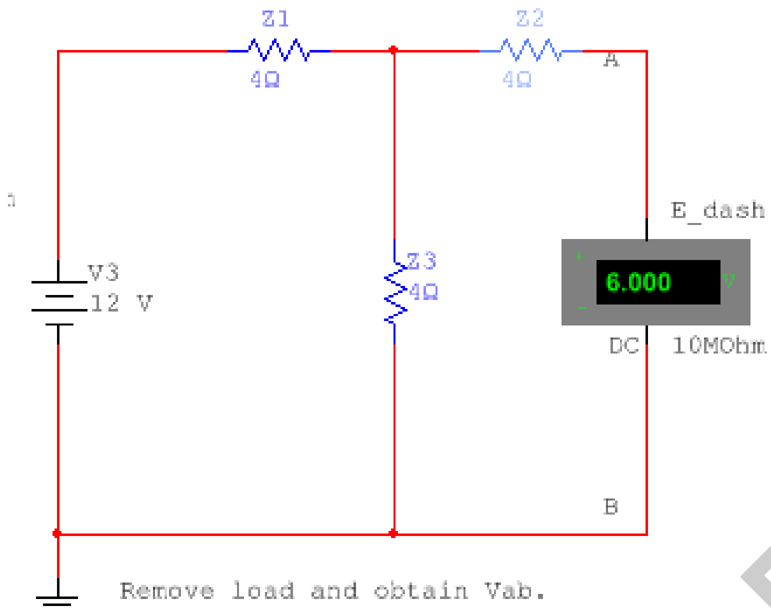
$$I_L = \frac{Z_3 E}{\{(Z_1 + Z_3)Z_2 + Z_1 Z_3 + (Z_1 + Z_3)Z_L\}} \Rightarrow I_L = \frac{\frac{Z_3}{(Z_1 + Z_3)}E}{\left\{\left(Z_2 + \frac{Z_1 Z_3}{(Z_1 + Z_3)}\right) + Z_L\right\}}$$

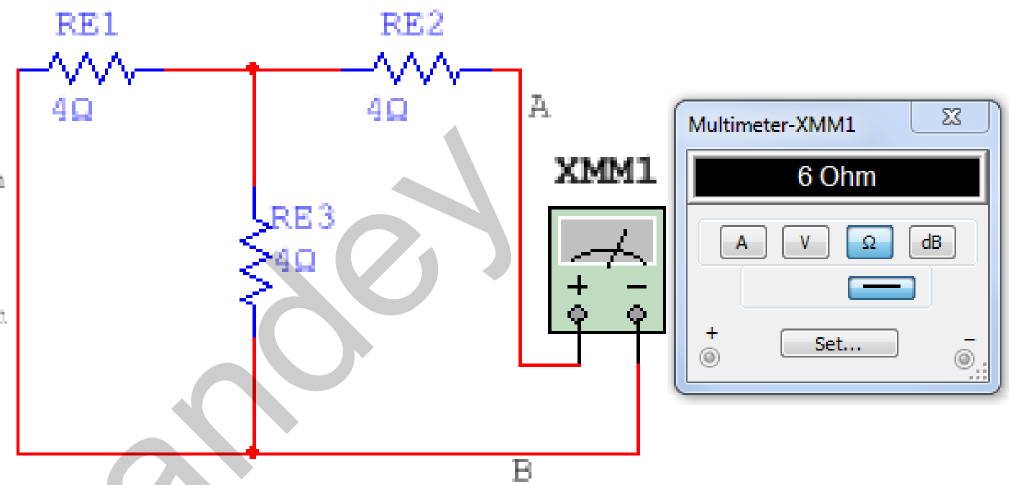
$$\boxed{I_L = \frac{E'}{Z' + Z_L}}$$

Find Load current using Thevenin's Theorem



Solution using Thevenin's Theorem

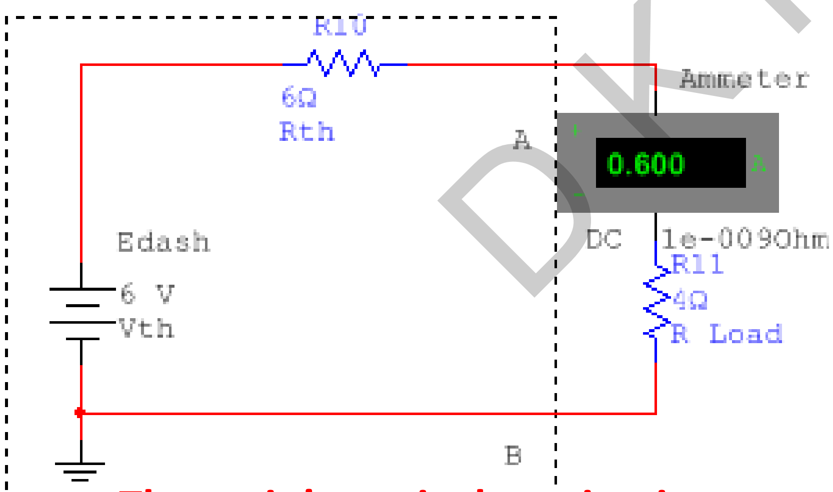




$$E' = \frac{Z_3}{(Z_1 + Z_3)} E$$

$$Z' = Z_2 + \frac{Z_1 Z_3}{(Z_1 + Z_3)}$$

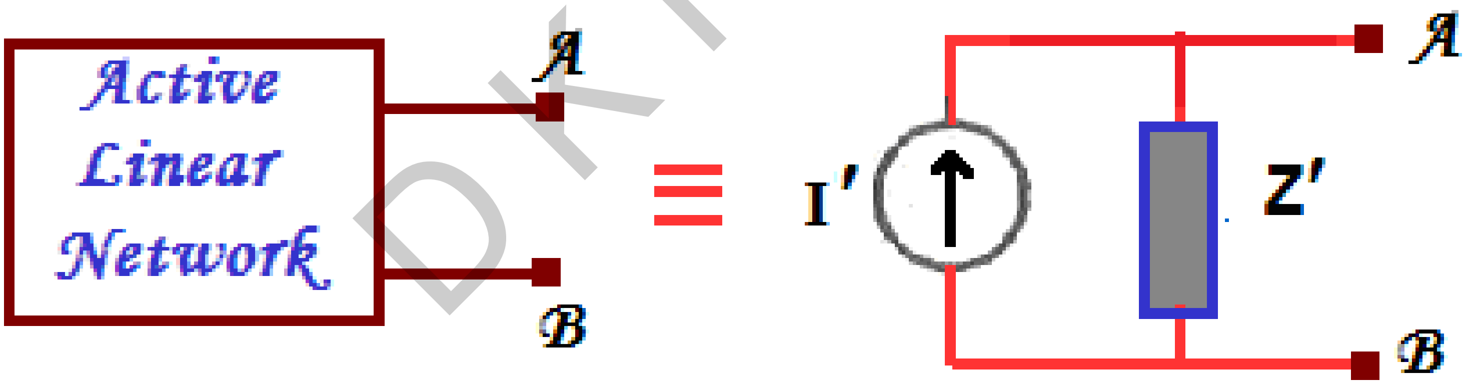
$$I_L = \frac{E'}{Z' + Z_L}$$



Thevenin's equivalent circuit

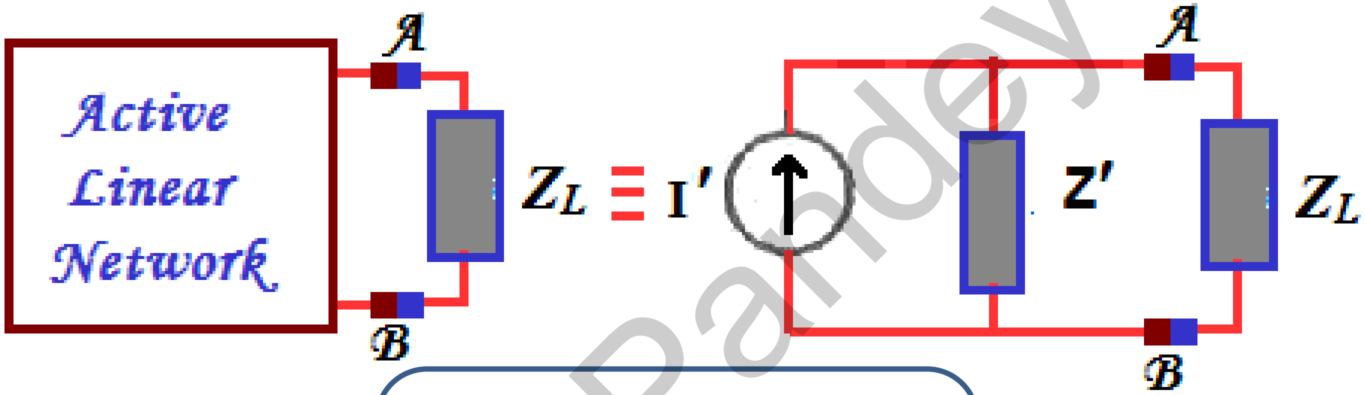
Norton's Theorem

Statement: *The theorem states that "Any two terminal linear network containing energy sources and impedances (active network) is equivalent to a current source of I′ in parallel combination with impedance Z′. Where I′* is short circuited current through terminals of network and *Z′* is the impedance of network when the sources are replaced by their internal impedances or short circuited.



Norton's Theorem ….. continued

If a load impedance Z_L *is connected to the terminal A and B of the network, then load current* I_L *can be written as,*



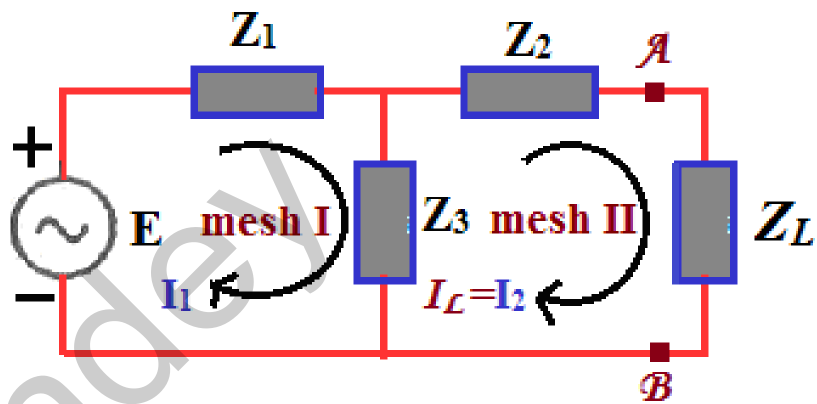
$$I_L = \frac{Z'I'}{Z' + Z_L} = \frac{I'}{1 + \frac{Z_L}{Z'}}$$

$$I' = \frac{Z_3 E}{(Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1)}$$

$$Z' = Z_2 + \frac{Z_1 Z_3}{(Z_1 + Z_3)}$$

Proof of Norton's Theorem

The mesh equation for mesh I and mesh II can be written as,



$$\boxed{\begin{aligned} Z_{11}I_1 + Z_{12}I_2 &= E_1 \\ Z_{21}I_1 + Z_{22}I_2 &= E_2 \end{aligned}}$$

Here,

$$Z_{11} = Z_1 + Z_3 \qquad Z_{12} = -Z_3 \qquad E_1 = E$$

$$Z_{21} = -Z_3 \qquad Z_{22} = Z_2 + Z_3 + Z_L \qquad E_2 = 0$$

So,

$$I_2 = \frac{\begin{bmatrix} Z_{11} & E_1 \\ Z_{21} & E_2 \end{bmatrix}}{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}} = \frac{\begin{bmatrix} Z_1 + Z_3 & E \\ -Z_3 & 0 \end{bmatrix}}{\begin{bmatrix} Z_1 + Z_3 & -Z_3 \\ -Z_3 & Z_2 + Z_3 + Z_L \end{bmatrix}} \Rightarrow I_2 = I_L = \frac{0 - (-Z_3)E}{\left\{(Z_1 + Z_3)(Z_2 + Z_3 + Z_L) - Z_3^2\right\}}$$

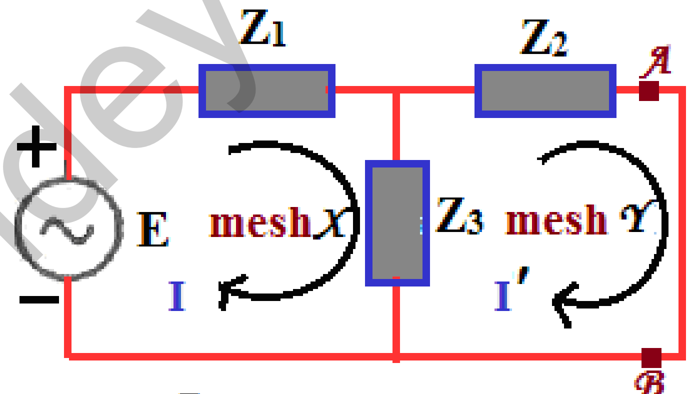
$$\Rightarrow I_L = \frac{Z_3 E}{\left\{Z_1Z_2 + Z_2Z_3 + Z_3Z_1 + (Z_1 + Z_3)Z_L\right\}} \rightarrow \boxed{I_L = \frac{\dfrac{Z_3 E}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}}{\left\{1 + \dfrac{Z_1 + Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} Z_L\right\}}}$$

Proof of Norton's Theorem ....continued

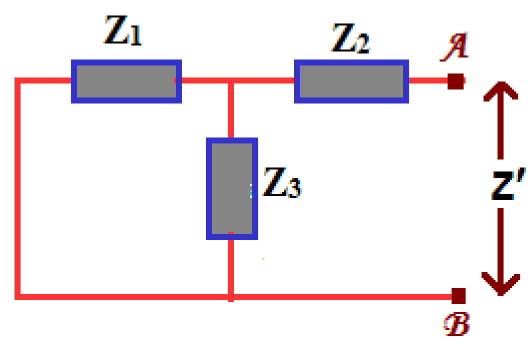
If I' is short circuited current through terminals of network and Z' is the impedance of network when the sources are short circuited then,

$$I' = \frac{\begin{bmatrix} Z_1 + Z_3 & E \\ -Z_3 & 0 \end{bmatrix}}{\begin{bmatrix} Z_1 + Z_3 & -Z_3 \\ -Z_3 & Z_2 + Z_3 \end{bmatrix}}$$

$$\boxed{I' = \frac{Z_3 E}{(Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1)}}$$



$$Z' = Z_2 + (Z_1 \parallel Z_3) \Rightarrow Z' = Z_2 + \frac{Z_1 Z_3}{(Z_1 + Z_3)}$$



$$\boxed{Z' = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1 + Z3}}$$

Thus,

$$I_L = \frac{I'}{1 + \frac{Z_L}{Z'}} \Rightarrow I_L = \frac{Z' I'}{Z' + Z_L}$$

Equivalence of Thevenin and Norton's Theorem

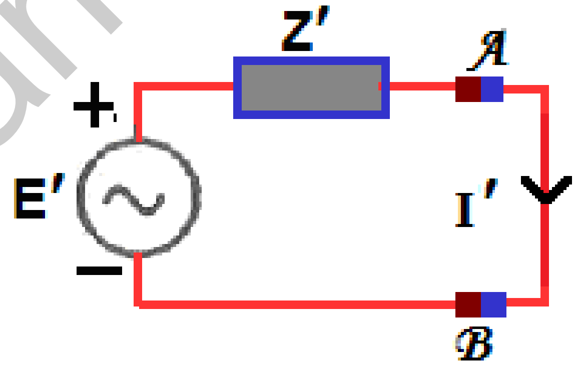
From *Thevenin* theorem, the load current can be written as,

$$I_L = \frac{E'}{Z' + Z_L}$$

If I' is shot circuited current through terminal of network, then

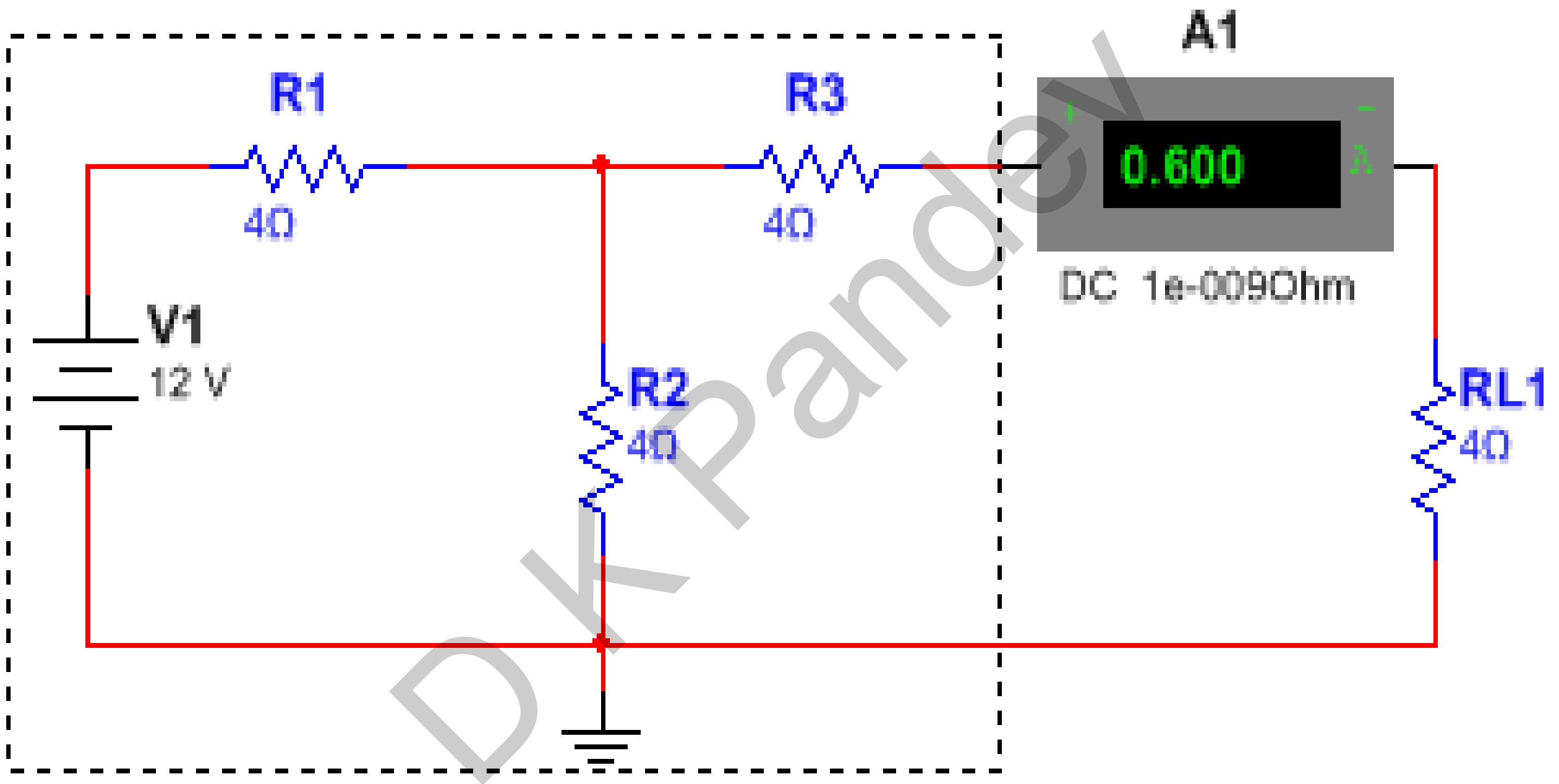
$$E' = Z'I'$$

$$I_L = \frac{I'Z'}{Z' + Z_L}$$

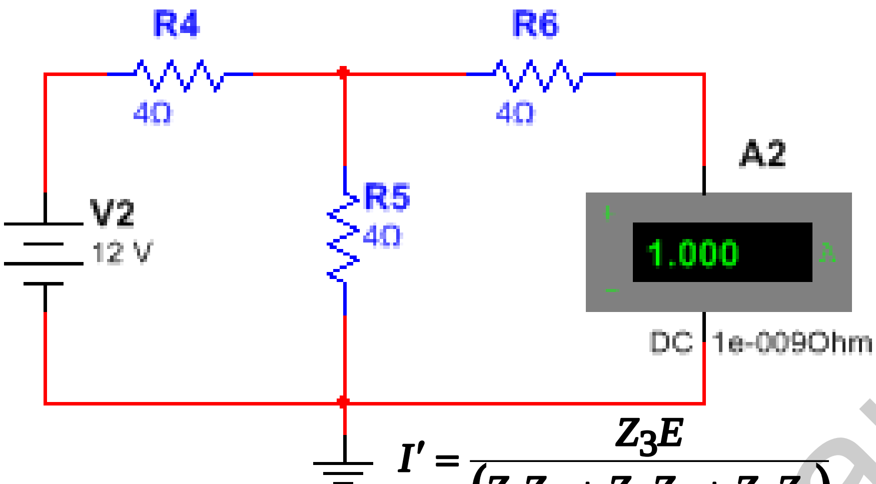


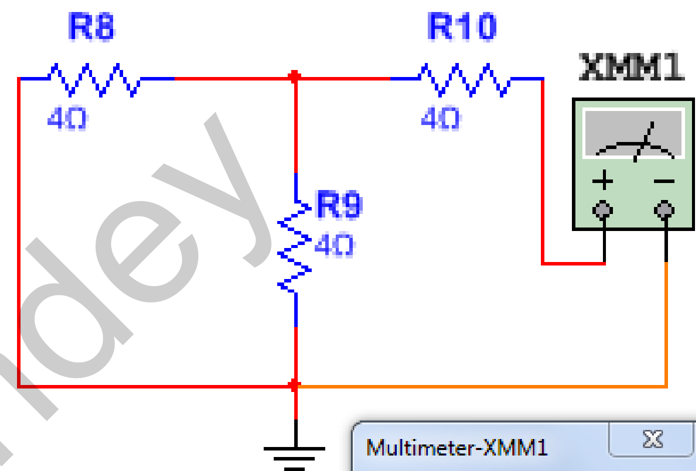
This is Nortons formula for Load current I_L. Therefore both the theorem are equivalent to each other.

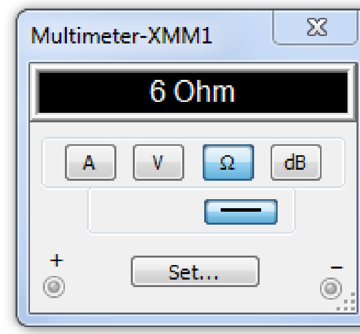
Find load current using Norton's Theorem



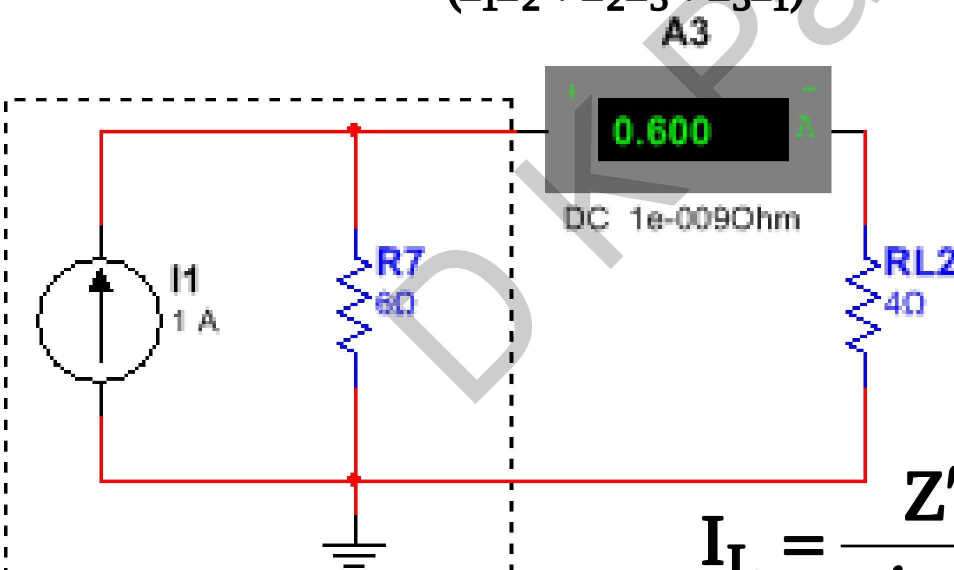
Solution using Norton's Theorem







$$I' = \frac{Z_3E}{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1)}$$



$$Z' = Z_2 + \frac{Z_1Z_3}{(Z_1 + Z_3)}$$

$$I_L = \frac{Z'I'}{Z' + Z_L}$$

Superposition Theorem

Statement: *The theorem states that "In any linear network containing impedances and energy sources (active linear network), the current in any element or branch or mesh is equal to algebraic sum of currents that would separately flow in that by each source while other sources are replaced by their internal impedances".*

$$I = \sum I_x \quad ; \quad x = 1,2,\ldots\ldots,n$$

Here n is number of energy sources in the network. If a two mesh active linear network has two energy sources and I_1 & I_2 are the currents in mesh1 and mesh 2 then,

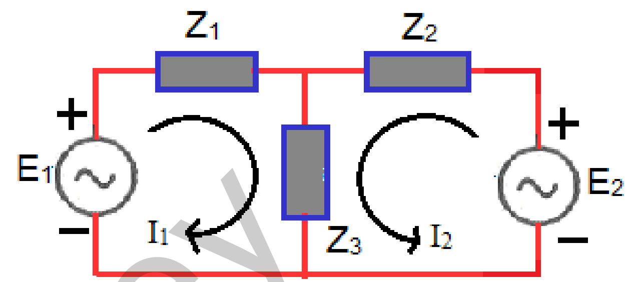
$$I_1 = I_1' + I_1'' \qquad\qquad I_2 = I_2' + I_2''$$

Where I_1' and I_2' are currents in mesh1 and mesh 2 by first energy source while I_1'' and I_2'' are currents in mesh1 and mesh 2 by second energy source.

Superposition Theorem

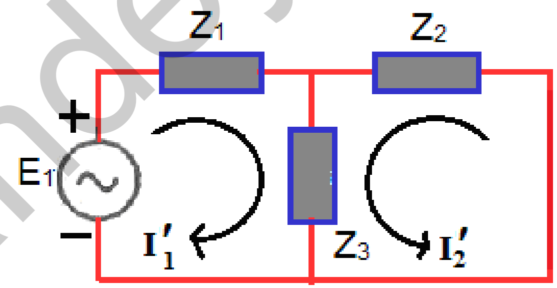
$$(Z_1 + Z_3)I_1 + Z_3 I_2 = E_1$$
$$Z_3 I_1 + (Z_2 + Z_3)I_2 = E_2$$

(1)



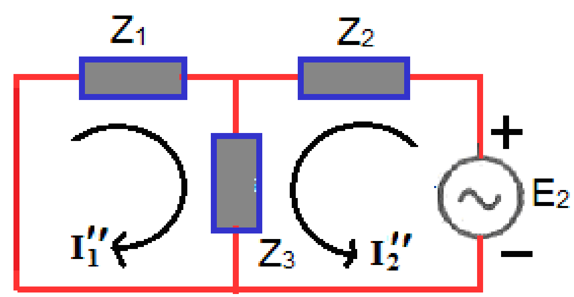
$$(Z_1 + Z_3)I_1' + Z_3 I_2' = E_1$$
$$Z_3 I_1' + (Z_2 + Z_3)I_2' = 0$$

(2)



$$(Z_1 + Z_3)I_1'' + Z_3 I_2'' = 0$$
$$Z_3 I_1'' + (Z_2 + Z_3)I_2'' = E_2$$

(3)



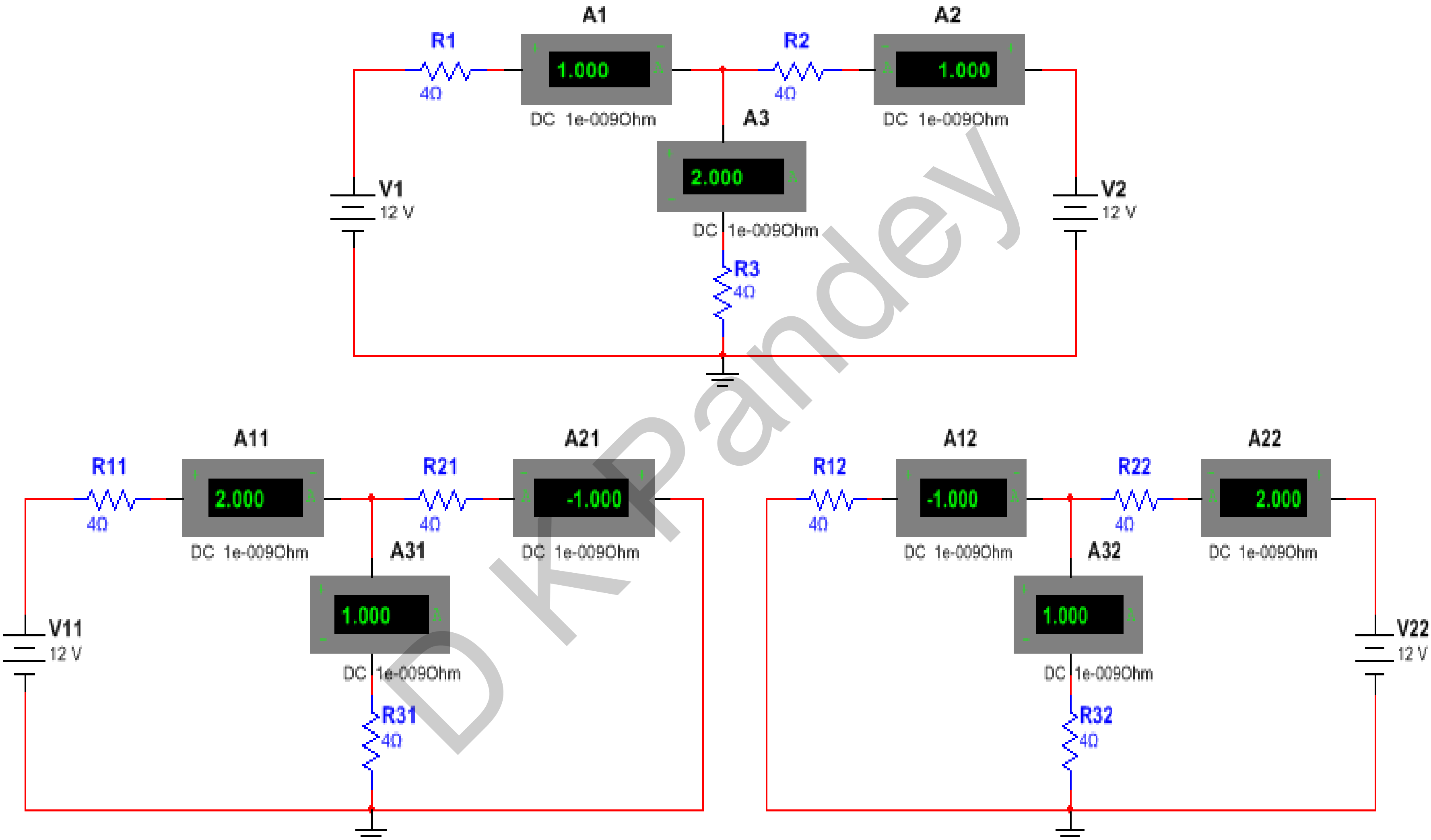
After (2)+ (3), we get

$$(Z_1 + Z_3)(I_1' + I_1'') + Z_3(I_2' + I_2'') = E_1$$
$$Z_3(I_1' + I_1'') + (Z_2 + Z_3)(I_2' + I_2'') = E_2$$

Comparing with (1)

$$I_1 = I_1' + I_1''$$
$$I_2 = I_2' + I_2''$$

Example of Superposition



A Lot of Thanks

for kind attention