Lecture 6,7&8: AC Bridges

A.C. Bridges

Note A: Alternating voltage: *An alternating voltage can be written as complex number. It is also known as phaser voltage.*

$$\mathbf{V} = V_0\, e^{j\omega t}$$

$$\mathbf{V} = V_0\, (\cos \omega t + j \sin \omega t)$$

$$\mathbf{V} = V_0 \cos \omega t + j\, V_0 \sin \omega t$$

Real part $V = V_0 \cos \omega t$

Img. part $V = V_0 \sin \omega t$

Here V_0 *=maximum or peak value of voltage*

$\theta = \omega t$ *=phase of a.c. voltage*

ω *=angular frequency of a.c. voltage*

Note B: Alternating current: *Similar to an alternating voltage, the alternating current can also be written as complex number. It is also known as phaser current.*

$$\mathbf{I} = I_0\, e^{j\omega t}$$

$$\mathbf{I} = I_0\, (\cos \omega t + j \sin \omega t)$$

$$\mathbf{I} = I_0 \cos \omega t + j\, I_0 \sin \omega t$$

Real part $I = I_0 \cos \omega t$

Img. part $I = I_0 \sin \omega t$

Here I_0 *=maximum or peak value of current*

Note C: Impedance: *Impedance is an important parameter used to characterize electronic circuits, components, and the materials used to make components.*

Impedance (Z) is generally defined as the total opposition a device or circuit offers to the flow of an alternating current at a given frequency. It is ratio of phaser voltage and phaser current. i.e.

$$Z = \mathbf{V}/\mathbf{I}$$

Let the phaser voltage leads the phaser current by an angle ϕ*. Then the equation for voltage and current will be:*

$$\mathbf{V} = V_0\, e^{j(\omega t + \phi)}$$

$$\mathbf{I} = I_0\, e^{j\omega t}$$

$$\text{Thus, } Z = \frac{V_0\, e^{j(\omega t+\phi)}}{I_0\, e^{j\omega t}} = \frac{V_0}{I_0} e^{j\phi}$$

$\boxed{Z = Z_0 e^{j\phi} = Z_0 \angle \phi}$ *: In polar co-ordinates*

$$Z = Z_0 \cos\phi + jZ_0 \sin\phi$$

$$\boxed{Z = R + jX}$$

Thus impedance is represented as a complex quantity. An impedance vector consists of a real part resistance, R and an imaginary part (reactance) X as shown in the Figure.

$R = Z_0 \cos\phi$

$X = Z_0 \sin\phi$

Magnitude of $Z = \sqrt{R^2 + X^2}$

Phase angle of Z; $\phi = \tan^{-1}(X/R)$

The reactances are of two ftypes:

Inductive reactance $X_L = \omega L = 2\pi f L$

Capacitive reactance $X_C = 1/\omega C = 1/(2\pi f C)$

Here f and ω are the frequency and angular frequency of a.c. source. L is inductance, and C is capacitance.

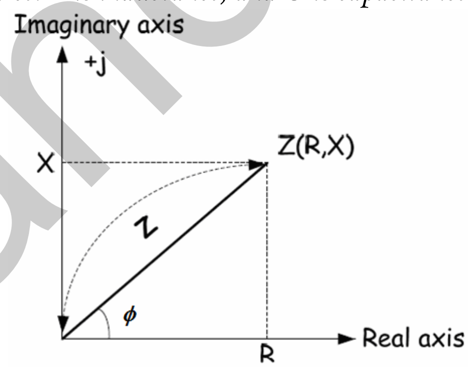


Figure A

The unit of impedance is the ohm (Ω). Impedance is a commonly used parameter and is especially useful for representing a series connection of resistance and reactance.

Note D: Admittance: *Admittance is ratio of phaser current and voltage or is equal to reciprocal of impedance. i.e.* $Y = 1/Z = \mathbf{I}/\mathbf{V}$

If phaser voltage and current are :

$$\mathbf{V} = V_0\, e^{j(\omega t+\phi)},\ \mathbf{I} = I_0\, e^{j\omega t}$$

$$\text{Then, } Y = \frac{1}{Z} = \frac{I_0\, e^{j\omega t}}{V_0\, e^{j(\omega t+\phi)}} = \frac{I_0}{V_0} e^{-j\phi}$$

$$\boxed{Y = Y_0 e^{-j\phi} = Y_0 \angle -\phi}$$

$$Y = Y_0 \cos(-\phi) + jY_0 \sin(-\phi)$$

$$\boxed{Y = 1/Z = G + jB}$$

Here G is conductance, and B is susceptance. The unit of admittance is the siemen (S).

Lecture 6,7&8: AC Bridges

A.C. Bridges: *AC bridges are similar to Wheatstone bridge in which D.C. source is replaced by an A.C. source and galvanometer with head phone/null detector. The resistors of bridge are replaced with combination of resistor, inductor and capacitors (i.e. impedances). These bridges are used to determine the unknown capacitance/inductance of capacitor/inductor. The working of these bridges is also based on Ohm's and Kirchoff's law.*

Circuit diagramme: *It has four arms with impedances forming a bridge. The two opposite junctions are connected to a head phone while the other two are connected with an a.c. source (Fig.1).*

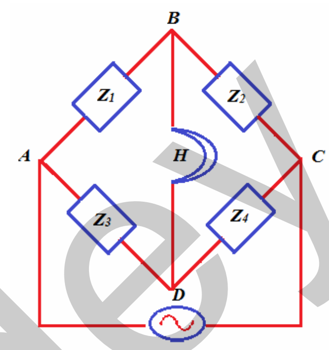


Fig.1

Principle and Balance condition: *When the potential difference across B and D becomes zero. No current flows through BD arm. Thus no sound is heard in head phone. This situation is called as bridge balance situation. The bridge balance condition can be obtained using the Kirchoff's voltage and current law. Suppose that, total current flowing from source is I. The currents in the arms AB, BC, AD, DC and BD are I_1, I_2, I_3, I_4 and I_H respectively.*

Since under the balance condition the head phone current (I_H) is zero. Thus,

$$I_1 = I_2 \quad \text{and} \quad I_3 = I_4$$

Applying KVL for loop ABDA, we have

$$Z_1 I_1 - Z_3 I_3 = 0$$

$$\Rightarrow \quad Z_1 I_1 = Z_3 I_3 \qquad (1)$$

And applying KVL for loop BCDB, we have

$$Z_2 I_2 - Z_4 I_4 = 0$$

$$\Rightarrow \quad Z_2 I_1 - Z_4 I_3 = 0$$

$$\Rightarrow \quad Z_2 I_1 = Z_4 I_3 \qquad (2)$$

Dividing eq.(1) with eq.(2) we have,

$$\frac{Z_1 I_1}{Z_2 I_1} = \frac{Z_3 I_3}{Z_4 I_3}$$

$$\Rightarrow \quad \boxed{\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}} \qquad (3a)$$

$$\text{Or} \quad \boxed{\frac{Z_1}{Z3} = \frac{Z_2}{Z_4}} \qquad (3b)$$

Eq.(3) is the balance condition for a.c bridges. This indicates that when the ratio of impedances in two adjacent arms of bridge is equal to the ratio of impedances of other two adjacent arms, then the bridge is balanced.

Since impedance is combination of real and imaginary part. Thus the above balance condition can be divided in to two sub condition.

Let, $Z_1 = R_1 + jX_1;\ Z_2 = R_2 + jX_2$

$Z_3 = R_3 + jX_3;\ Z_4 = R_4 + jX_4$

Putting impedances in eq.(3b)

$$\frac{R_1 + jX_1}{R_3 + jX_3} = \frac{R_2 + jX_2}{R_4 + jX_4}$$

$$(R_1 + jX_1)(R_4 + jX_4) = (R_2 + jX_2)(R_3 + jX_3)$$

$$R_1R_4 - X_1X_4 + j(R_1X_4 + R_1X_4)$$

$$= R_2R_3 - X_2X_3 + j(R_2X_3 + R_3X_2) \quad (4)$$

Comparing real term of eq.(4) we have,

$$R_1R_4 - X_1X_4 = R_2R_3 - X_2X_3 \qquad (5)$$

If $\quad X_1X_4 = X_2X_3 \qquad (6)$

Under the condition given by eq.(6), the eq.(5) becomes as,

$$R_1R_4 = R_2R_3 \qquad (7a)$$

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}} \qquad (7b)$$

Eq.(7) is the real part condition of the bridge balance condition.

Comparing imaginary terms of eq.(5), we have,

$$R_1X_4 + R_4X_1 = R_2X_3 + R_3X_2 \qquad (8)$$

Dividing eq.(8) by eq.(7b), we get,

$$\frac{R_1X_4 + R_4X_1}{R_1R_4} = \frac{R_2X_3 + R_3X_2}{R_2R_3}$$

$$\frac{X_4}{R_4} + \frac{X_1}{R_1} = \frac{X_3}{R_3} + \frac{X_2}{R_2}$$

$$\frac{X_1}{R_1}+\frac{X_4}{R_4}=\frac{X_2}{R_2}+\frac{X_3}{R_3}$$

$$\boxed{\phi_1+\phi_4=\phi_2+\phi_3} \qquad (9)$$

Here ϕ is phase angle. The eqs. (6), (7) and (9) suggests that a.c. bridge balance condition has two type of balance condition. i.e.(i) magnitude and (ii) phase angle balance condition.

Alternate proof of balance condition : *AC bridge balance condition is obtained when,*

$$V_B=V_D$$

If V is the source voltage then,

$$\frac{Z_1}{Z_1+Z_2}V=\frac{Z_3}{Z_3+Z_4}V$$

$$\frac{Z_1}{Z_1+Z_2}=\frac{Z_3}{Z_3+Z_4}$$

$$\frac{Z_1+Z_2}{Z_1}=\frac{Z_3+Z_4}{Z_3}$$

$$\frac{Z_2}{Z_1}=\frac{Z_4}{Z_3} \text{ or } \boxed{\frac{Z_1}{Z_2}=\frac{Z_3}{Z_4}} \qquad (1)$$

This is the ac bridge balance condition.

Since $Z = Z_0\angle\phi$; Z_0=magnitude of Z

Thus, $Z_1=(Z_0)_1\angle\phi_1,\ Z_2=(Z_0)_2\angle\phi_2$

$Z_3=(Z_0)_3\angle\phi_3,\ Z_4=(Z_0)_4\angle\phi_4$

Putting values of impedances in eq.(1),

$$\frac{(Z_0)_1\angle\phi_1}{(Z_0)_2\angle\phi_2}=\frac{(Z_0)_3\angle\phi_3}{(Z_0)_1\angle\phi_4}$$

$$\frac{(Z_0)_1}{(Z_0)_2}\angle(\phi_1-\phi_2)=\frac{(Z_0)_3}{(Z_0)_1}\angle(\phi_3-\phi_4)$$

$$\frac{(Z_0)_1}{(Z_0)_2}\angle(\phi_1+\phi_4)=\frac{(Z_0)_3}{(Z_0)_1}\angle(\phi_2+\phi_3) \qquad (2)$$

Separating the magnitude and phase angle in Eq.(2), we have

$$\frac{(Z_0)_1}{(Z_0)_2}=\frac{(Z_0)_3}{(Z_0)_1} \qquad (3a)$$

$$(\phi_1+\phi_4)=(\phi_2+\phi_3) \qquad (3b)$$

Eqs. (3a) and (3b) suggests that a.c. bridge balance condition has two type of balance condition. i.e.(i) magnitude and (ii) phase angle balance condition.

Parameters measured from AC Bridges: *The capacitance of capacitor, inductance of an inductor and frequency of ac source can be measured with AC bridge.*

AC bridges for measurement of C

(1) *De-sauty bridge*
(2) *Weins bridge (Series)*
(3) *Schering bridge*

AC bridges for measurement of L

(4) *Anderson bridge*
(5) *Maxwell inductance bridge*
(6) *Maxwell L/C bridge or Maxwell-Weins bridge*
(7) *Hay bridge*
(8) *Owen's bridge*
(9) *Heavisible –Campbell equal ratio bridge*

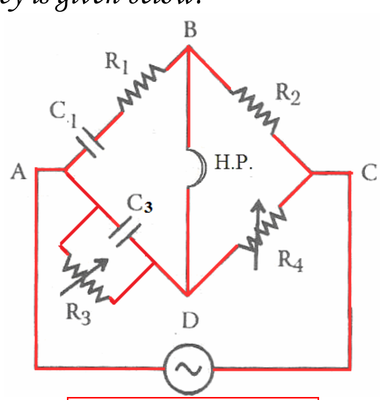
AC bridges for measurement of 'f'

(10) *Robinson bridge*
(11) *Weins bridge (parallel)*

Note: There are two types of Weins bridge:

(a) *Weins series bridge: It is used to determine the unknown capacitance, its power factor .*

(b) *Weins parallel bridge: It is used in feedback network circuit of oscillator. It is used as frequency determining element in audio and high frequency oscillators. It is also used in harmonic distortion analyzer, where it is used as a notch filter for discriminating against one specific frequency. The circuit diagram and formula for frequency is given below.*



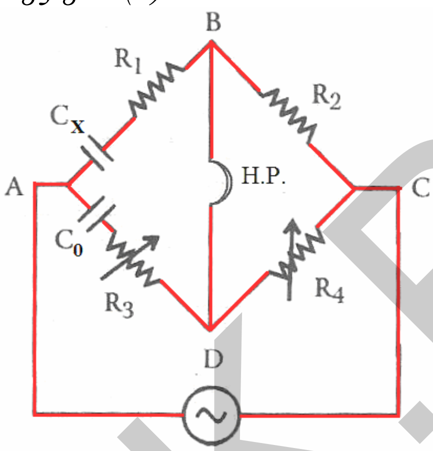
$$\boxed{f=\frac{1}{2\pi\sqrt{C_1C_3R_1R_3}}}$$

If $R_1=R_3$ *and* $C_1=C_3$ *then* $R_2=2R_4$ *and* $\boxed{f=1/(2\pi RC)}$

Wein's Bridges: *It is an ac bridge. It is used to determine the unknown capacitance of a capacitor in terms of known capacitance of standard capacitor and is used to define the quality of capacitor by determination of its power factor.*

The four arms of this encloses following components.

AB arm: *Capacitor of unknown capacitance* C_x *with a series internal resistance* R_1
BC arm: *Fixed resistance* R_2
AD arm: *Standard capacitor of known capacitance* C_0 *with a series non-inductive variable resistance* R_3
DC arm: *Variable resistance* R_4
The complete bridge circuit is shown in following figure (X).



Figure(X)

Working: The variable resistances R_3 *and* R_4 *are varied at fixed value of* R_2*, till no sound is heard in head phone. At no sound, bridge becomes balanced.*
Let Z_1, Z_2, Z_3 *and* Z_4 *are the impedances of the four arms of the bridge. Then,*

$$Z_1 = R_1 + \frac{1}{j\omega C_X}\ ;\ Z_2 = R_2$$

$$Z_3 = R_3 + \frac{1}{j\omega C_0}\ ;\ Z_4 = R_4$$

Under the balance condition of bridge,

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} \qquad (1)$$

Putting values of impedances, we have

$$\frac{R_1 + \frac{1}{j\omega C_X}}{R_2} = \frac{R_3 + \frac{1}{j\omega C_0}}{R_4}$$

$$\frac{R_1}{R_2} + \frac{1}{j\omega C_X R_2} = \frac{R_3}{R_4} + \frac{1}{j\omega C_0 R_4} \qquad (2)$$

Comparing real part of eq.(2), we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad (3)$$

Eq.(3) is the real part/dc balance condition of this bridge. By this, the series internal resistance of unknown capacitor can be determined.
And comparing imaginary part of eq.(2), we have

$$\frac{1}{\omega C_X R_2} = \frac{1}{\omega C_0 R_4}$$

$$C_X R_2 = C_0 R_4$$

$$\boxed{C_X = C_0 \frac{R_4}{R_2}} \qquad (4)$$

The equation (4) is the formula for the determination of unknown capacitance. On the knowledge of R_2 *and* R_4 *in balance condition, the unknown capacitance is calculated with eq.(4).*
The power factor of unknown capacitor can be written as,

$$\cos\phi = \frac{R_1}{Z} = \frac{R_1}{\sqrt{R_1^2 + (1/\omega C_x)^2}}$$

If $R_1 < (1/\omega C_x)$ *then,*

$$\boxed{\cos\phi = \omega C_x R_1} \qquad (5)$$

By knowing the value of R_1 *and* C_X*, the power factor can be determined with eq.(5). If power factor is small then quality of capacitor is good otherwise not.*

Advantage

This bridge is most suitable for comparing capacitances of capacitors.

Disadvantage

In this bridge, final balance condition is obtained by alternate variation in R_3 *and* R_4*. Thus both the balance conditions are dependent on each other. Thus sensitivity of bridge is low and is high for equal value of* R_2 *and* R_4*.*

Schering Bridges: *This is an ac bridge which is used for the determination of the most accurate value of the unknown capacitance of a capacitor and is used to define the quality of capacitor by determination of its power factor. This bridge is also used in measurement of dielectric constants of liquids, testing of cables and insulators at high voltages.*

The four arms of this encloses following components.

AB arm: *Capacitor of unknown capacitance* C_1 *with a series internal resistance* R_1

BC arm: *Fixed resistance* R_2

AD arm: *Standard Capacitor of known capacitance* C_0

DC arm: *A variable capacitor of capacitance* C_4 *in parallel combination with Variable resistance* R_4.

The complete bridge circuit is shown in following figure (Y).

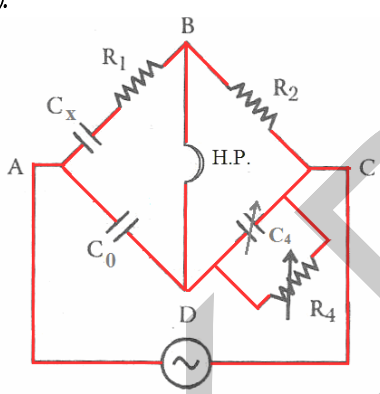


Figure (Y)

Working: The variable capacitor C_4 *and resistance* R_4 *are varied at fixed value of* R_2, *till no sound is heard in head phone. At no sound, bridge becomes balanced.*

Let Z_1, Z_2, Z_3 *and* Z_4 *are the impedances of the four arms of the bridge. Then,*

$$Z_1 = R_1 + \frac{1}{j\omega C_X};\ Z_2 = R_2$$

$$Z_3 = \frac{1}{j\omega C_0};\ \frac{1}{Z_4} = \frac{1}{R_4} + j\omega C_4$$

Under the balance condition of bridge,

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} \qquad (1)$$

Putting values of impedances, we have

$$\frac{R_1 + \frac{1}{j\omega C_X}}{R_2} = \frac{1}{j\omega C_0}\left(\frac{1}{R_4} + j\omega C_4\right)$$

$$\frac{R_1}{R_2} + \frac{1}{j\omega C_X R_2} = \frac{C_4}{C_0} + \frac{1}{j\omega C_0 R_4} \qquad (2)$$

Comparing real part of eq.(2), we have

$$\frac{R_1}{R_2} = \frac{C_4}{C_0} \qquad (3)$$

And comparing imaginary part of eq.(2), we have

$$\frac{1}{\omega C_X R_2} = \frac{1}{\omega C_0 R_4} \qquad \Rightarrow C_X R_2 = C_0 R_4$$

$$C_X = C_0 \frac{R_4}{R_2} \qquad (4)$$

Eq.(3) is the first balance condition of this bridge. By this, the series internal resistance of unknown capacitor can be determined. This condition also indicates that the variation in C_4 *results this balance condition and is independent of* R_4.

The equation (4) is second balance condition and provides formula for the determination of unknown capacitance. On the knowledge of R_2 *and* R_4 *in balance condition, the unknown capacitance is calculated with eq.(4). This equation also suggests that this balance is obtained by variation in* R_4 *and is independent of* C_4.

Thus C_4 *and* R_4 *are varied at fixed value of* R_2 *for getting the balance condition.*

The power factor of unknown capacitor can be written as,

$$\cos\phi = \frac{R_1}{Z} = \frac{R_1}{\sqrt{R_1^2 + (1/\omega C_x)^2}} \qquad (5)$$

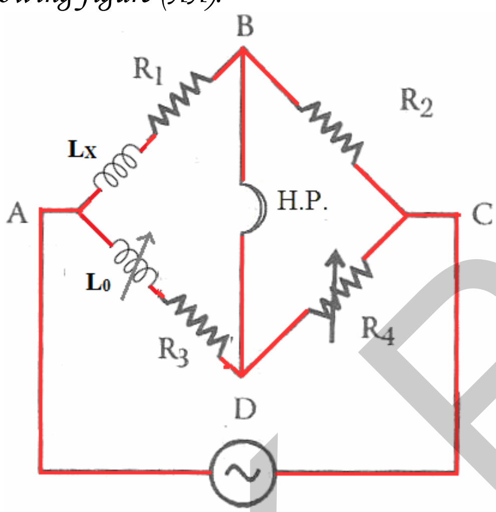
Power factor of unknown capacitor can be determined with eq.(5) by knowing the value of R_1 *and* C_X. *Since the Both* C_4 *and* R_4 *are required for determination* R_1 *and* C_X, *thus power factor require both the variable quantity. If power factor is small then quality of capacitor is good otherwise not.*

Advantage: This bridge provides Good/fine balance condition, most accurate result and is most sensitive. It is also useful in measurement of dielectric constants of liquids and testing of cables and insulators at high voltage.

Maxwell inductance bridges: *It is the simplest ratio AC Bridge for the determination of unknown medium inductance of an inductor. This bridge is very similar to Weins series bridge.*

The four arms of this bridge encloses following components.

AB arm: *Inductor of unknown inductance* L_x *with a series internal resistance* R_1
BC arm: *Fixed resistance* R_2
AD arm: *Variable inductor of known inductance* L_0 *with its series internal resistance* R_3
DC arm: *Variable resistance* R_4
The complete bridge circuit is shown in following figure (XX).



Figure(XX)

Working: The variable inductance L_0 *and resistance* R_4 *are varied at fixed value of* R_2*, till no sound is heard in head phone. At no sound, bridge becomes balanced.*

Let Z_1, Z_2, Z_3 *and* Z_4 *are the impedances of the four arms of the bridge. Then,*

$Z_1 = R_1 + j\omega L_X$; $Z_2 = R_2$

$Z_3 = R_3 + j\omega L_0$; $Z_4 = R_4$

Under the balance condition of bridge,

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} \qquad (1)$$

Putting values of impedances, we have

$$\frac{R_1 + j\omega L_X}{R_2} = \frac{R_3 + j\omega L_0}{R_4}$$

$$\frac{R_1}{R_2} + j\omega\frac{L_X}{R_2} = \frac{R_3}{R_4} + j\omega\frac{L_0}{R_4} \qquad (2)$$

Comparing real part of eq.(2), we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad (3)$$

Eq.(3) is the real part/dc balance condition of this bridge. By this, the series internal resistance of unknown inductor can be determined.

And comparing imaginary part of eq.(2), we have

$$\frac{L_X}{R_2} = \frac{L_0}{R_4}$$

$$\boxed{L_X = L_0\frac{R_2}{R_4}} \qquad (4)$$

The equation (4) is the formula for the determination of unknown inductance. On the knowledge of R_2 *and* R_4 *in balance condition, the unknown inductance is calculated.*

Advantage

From eqs. (3) and (4), we can write,

$$L_X = L_0\frac{R_1}{R_3}$$

$$\frac{L_X}{R_1} = \frac{L_0}{R_3} \qquad (5)$$

Thus balance condition is obtained when,
Time constant of unknown inductor
=time constant of known inductor

Hence, this bridge is the most suitable for the comparing inductances and in measurement of self inductance in terms of known self inductance. This bridge is also used for measuring the iron losses of the transformers at audio frequency.

Disadvantage

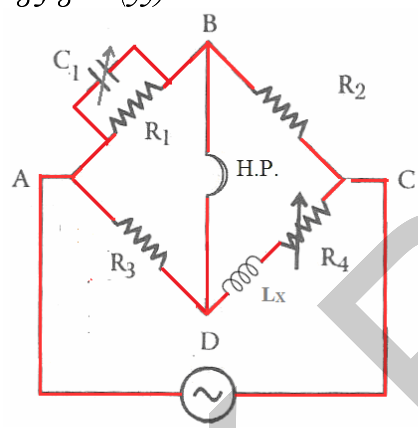
The disadvantage of this bridge is that the both balance condition can not be satisfied independently because any change in L_0 *causes change in* R_3*. Thus process of getting balance is not easy.*

Lecture 6,7&8: AC Bridges

Maxwell L/C bridges: *This is an AC bridge which is also known as Maxwell-Wein bridge. It is modified Maxwell's inductance bridge. By this bridge unknown inductance of an inductor is measured in terms of capacitance.*

The four arms of this bridge encloses following components.

AB arm: *A variable capacitor of capacitance* C_1 *in parallel combination of resistance* R_1
BC arm: *Fixed resistance* R_2
AD arm: *Fixed resistance* R_3
DC arm: *An inductor of unknown inductance* L_X *in series of variable resistance* R_4
The complete bridge circuit is shown in following figure (yy).



Figure(yy)

Working: The variable inductance C_1 *and resistance* R_4 *are varied at fixed value of* R_1, R_2 *and* R_3, *till no sound or minimum sound is heard in head phone. At this situation, bridge becomes balanced.*

Let Z_1, Z_2, Z_3 *and* Z_4 *are the impedances of the four arms of the bridge. Then,*

$$\frac{1}{Z_1} = \frac{1}{R_1} + j\omega C_1; \; Z_2 = R_2$$

$$Z_3 = R_3 \qquad ; \; Z_4 = R_4 + j\omega L_X$$

Under the balance condition of bridge,

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$\frac{Z_2}{Z_1} = \frac{Z_4}{Z_3} \qquad (1)$$

Putting values of impedances, we have

$$R_2\left(\frac{1}{R_1} + j\omega C_1\right) = \frac{R_4 + j\omega L_X}{R_3}$$

$$\frac{R_2}{R_1} + j\omega C_1 R_2 = \frac{R_4}{R_3} + j\omega\frac{L_X}{R_3} \qquad (2)$$

Comparing real part of eq.(2), we have

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad (3)$$

Eq.(3) is the real part/dc balance condition of this bridge. Thus variation in R_4 *provides dc balance condition.*

And comparing imaginary part of eq.(2), we have

$$C_1 R_2 = \frac{L_X}{R_3}$$

$$\boxed{L_X = C_1 R_2 R_3} \qquad (4)$$

The equation (4) is the formula for the determination of unknown inductance. On the knowledge of C_1, R_2 *and* R_3 *in balance condition, the unknown inductance is calculated.*

Advantage

Both the balance conditions are independent to each other. Initially, R_4 *is varied then* C_1 *is varied to obtain final balance condition. Thus process of getting the balance condition is easy. In view of getting the balance condition, this bridge is better than the Maxwell's inductance bridge.*

Disadvantage

The perfect balance can never be obtained in this bridge due to stray capacitance (self capacitance of coil) and presence of harmonics in ac source.

