Lecture 2: Current in RC circuit

Charging of Capacitor through Resister

Let us consider a capacitor of capacitance C is connected to a DC source of e.m.f. E through a resister of resistance R and a key K in series. When the key K is switched on, the charging process of capacitor starts. The charge on capacitor increases with time and attains its maximum in certain duration of time. As the time passes, potential difference across the plates of capacitor increases due to increase of charge. This potential difference opposes the source for charging. As a result flow of charge reduces. Hence in the process of charging of capacitor, initially the maximum current flows through circuit and it decreases with time.

According to KVL, the algebraic sum of instantaneous voltage drop across the circuit elements for a closed loop is zero. Thus, for the present RL circuit,

$$E - V_1 - V_2 = 0$$

$$E = V_1 + V_2 \qquad (1)$$

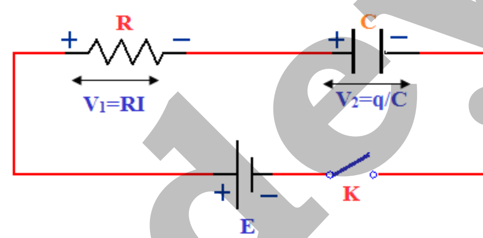

Fig 1

If at any instant the current in circuit is I and charge stored on capacitor is q then the potential drop across resister and capacitor will be RI and q/C respectively.

Using eq.(1), we can write,

$$E = RI + \frac{q}{C} \quad \Rightarrow RI = E - \frac{q}{C}$$

$$R\frac{dq}{dt} = \frac{CE - q}{C}$$

$$\frac{dq}{CE - q} = \frac{1}{RC}dt \qquad (2)$$

Integrating eq.(2)

$$\int \frac{dq}{CE - q} = \frac{1}{RC}\int dt + C'$$

$$-\log_e(CE - q) = \frac{1}{RC}t + C' \qquad (3)$$

Here C' is integration constant and is determined by initial condition.

i.e. when t=0, q=0 then from eq.(3) we have

$$-\log_e(CE) = C' \qquad (4)$$

From eqs.(3) and (4), we can write,

$$-\log_e(CE - q) = \frac{1}{RC}t - \log_e(CE)$$

$$\log_e(CE - q) = -\frac{1}{RC}t + \log_e(CE)$$

$$\log_e(CE - q) - \log_e(CE) = -\frac{1}{RC}t$$

$$\log_e\frac{(CE - q)}{(CE)} = -\frac{t}{RC}$$

$$\frac{CE - q}{CE} = e^{-\frac{t}{RC}}$$

$$1 - \frac{q}{CE} = e^{-\frac{t}{RC}}$$

$$\frac{q}{CE} = 1 - e^{-\frac{t}{RC}}$$

$$\boxed{q = CE\left(1 - e^{-\frac{t}{RC}}\right)} \qquad (5)$$

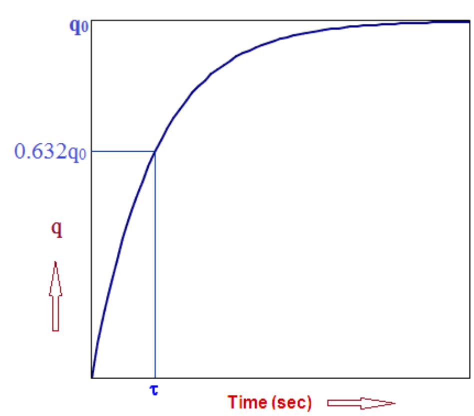

Fig.2

Lecture 2: Current in RC circuit

Since , the steady state charge or maximum charge (q_0)= CE
And dimension of RC = dimension of time;
Say, RC = τ = capacitive time constant
Hence eq.(4) becomes as,

$$\boxed{q = q_0\left(1 - e^{-t/\tau}\right)} \qquad (6)$$

Eqs. (5) and (6) are called as expression of charge on capacitor at instant t. These expressions indicates that-

(1) *Initially the charge on capacitor is zero.*
(2) *It increases exponentially following expression* $(1 - e^{-t/\tau})$.
(3) *After infinite time, it reaches to its steady state value (q_0).*

If V is potential difference across the plates of capacitor at instant t then from eq.(6)

$$CV = CE\left(1 - e^{-t/\tau}\right)$$

$$\boxed{V = E\left(1 - e^{-t/\tau}\right) = V_0\left(1 - e^{-t/\tau}\right)} \qquad (7)$$

Eq. (7) shows that the nature of potential difference across the plates of capacitor is similar to that of charge stored in capacitor.
Differentiating eq.(6),

$$\frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau}$$

$$\frac{dq}{dt} = \frac{CE}{RC} e^{-t/\tau}$$

$$\frac{dq}{dt} = \frac{E}{R} e^{-t/\tau}$$

$$\boxed{I = I_0\, e^{-t/\tau}} \qquad (8)$$

Eq. (8) is called as expression of current in the process of charging of capacitor. This expression indicates that-

(1) *Initially, the current through circuit is maximum (I_0).*
(2) *It decreases exponentially following expression* $(e^{-t/\tau})$.
(3) *After infinite time, it reaches to zero.*

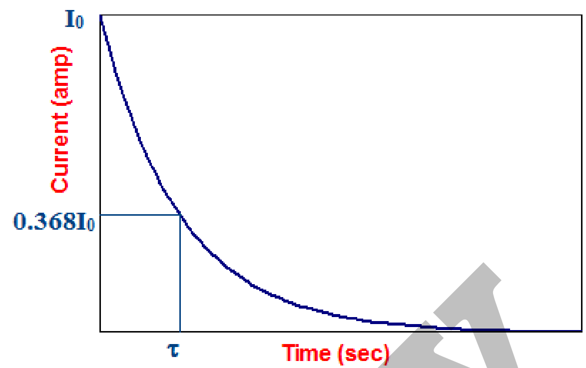

Fig.3

Capacitive Time constant

A. *We know that*

$$q = q_0\left(1 - e^{-t/\tau}\right)$$

When t=τ then from eq. (6), we can write

$$q = q_0\left(1 - e^{-1}\right)$$

$$q = q_0\left(1 - \frac{1}{e}\right)$$

$$q = q_0\left(1 - \frac{1}{2.718}\right)$$

$$q = q_0(1 - 0.368)$$

$$q = 0.632\, q_0$$

$$\boxed{q = 0.632\, q_0 = 63.2\% q_0}$$

Thus the time constant for the RC circuit is the time in which the current increases up to 63.2% of maximum current.

B. *We know that*

$$I = I_0\, e^{-t/\tau}$$

When t=τ then from eq. (6), we can write

$$I = I_0\, e^{-1} = I_0 \frac{1}{e} = I_0 \frac{1}{2.718} = 0.368\, I_0$$

$$\boxed{I = 0.368\, I_0 = 36.8\% I_0}$$

Thus the time constant for the RC circuit is the time in which the current decays from its maximum value to 36.8% of maximum current.

Lecture 2: Current in RC circuit

Discharging of capacitor through resister

Let us consider a charged capacitor of capacitance C is connected to a resister of resistance R through a key K in series. When the key K is switched on, the capacitor discharges through resister. The charge on capacitor started to decrease due to loss of capacitive energy through resister.

According to KVL, the algebraic sum of instantaneous voltage drop across the circuit elements for a closed loop is zero.

Thus $E - V_1 - V_2 = 0$

$E = V_1 + V_2$

Here $E=0$, *Thus* $V_1 + V_2 = 0$ *(1)*

If the current in circuit at time t is I and charge stored on capacitor is q then the potential drop across resister and capacitor will be RI and q/C respectively.

Using eq.(1)

$$RI + \frac{q}{C} = 0$$

$$\Rightarrow RI = -\frac{q}{C}$$

$$\Rightarrow \frac{dq}{dt} = -\frac{q}{RC}$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{RC}dt \qquad (2)$$

Integrating eq.(1)

$$\int \frac{dq}{q} = -\frac{1}{RC}\int dt + C'$$

$$\log_e q = -\frac{t}{RC} + C' \qquad (3)$$

Here C' *is integration constant and is determined by initial condition.*

i.e. when t=0, $q=q_0$ *then from eq.(3) we have*

$$\log_e q_0 = C' \qquad (4)$$

From eqs. (3) and (4), we can write,

$$\log_e q = -\frac{t}{RC} + \log_e q_0$$

$$\log_e \frac{q}{q_0} = -\frac{t}{RC} \qquad \Rightarrow \frac{q}{q_0} = e^{-t/RC}$$

$$\boxed{q = q_0\, e^{-t/RC}} \qquad (5)$$

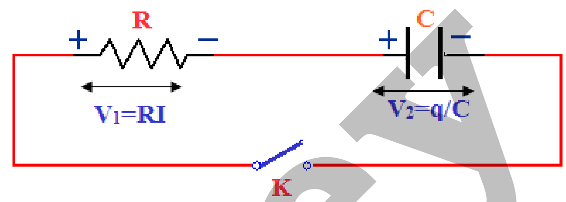

Fig.1

Since, dimension of RC = dimension of time;

Say, $RC = \tau$ = *Capacitive time constant*

Hence eq.(5) becomes as,

$$\boxed{q = q_0\, e^{-t/\tau}} \qquad (6)$$

Eqs. (5) and (6) are expressions for charge on capacitor at instant t in case of discharging. These expressions indicate that Initially the charge on capacitor has maximum value q_0 *and It decreases exponentially following expression* $(e^{-t/\tau})$.

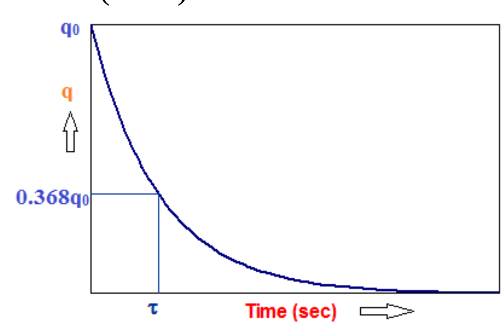

Fig.2

If V is potential difference across the plates of capacitor at instant t then from eq.(6)

$$CV = CE\, e^{-t/\tau}$$

$$\boxed{V = E\, e^{-t/\tau} = V_0\, e^{-t/\tau}} \qquad (7)$$

Eq. (7) is expression of potential difference across the plates of capacitor in its discharging case.

Lecture 2: Current in RC circuit

Differentiating eq.(6)

$$\frac{dq}{dt} = -\frac{q_0}{\tau} e^{-t/\tau} = -\frac{CE}{RC} e^{-t/\tau}$$

$$\frac{dq}{dt} = -\frac{E}{R} e^{-t/\tau}$$

$$\boxed{I = -I_0\, e^{-t/\tau}} \qquad (8)$$

Eq. (8) is called as expression of current in the process of discharging of capacitor. This expression indicates that-

(1) Initially, the maximum current flows through circuit in opposite direction of charging case.

(2) It decreases exponentially following expression$(e^{-t/\tau})$.

(3) After infinite time, it approaches to zero.

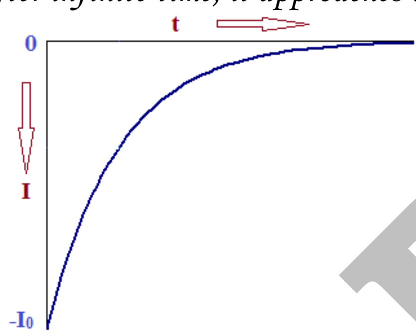

Fig.3

Capacitive Time Constant

In case of discharging of capacitor, the charge on capacitor at instant is given by following expression.

$$q = q_0\, e^{-t/RC}$$

When t=τ then from above eq., we can write

$$q = q_0\, e^{-1} = q_0 \frac{1}{e} = q_0 \frac{1}{2.718} = 0.368\, q_0$$

$$\boxed{q = 0.368\, q_0 = 36.8\%\, q_0}$$

Thus the time constant for the RC circuit is the time in which the charge on capacitor decays from steady state value to 36.8% of its maximum.

High resistance by leakage of capacitor

The high resistance can be measured with the discharging process of capacitor.

Principle: In the case of discharging of capacitor, the charge 'q' on capacitor at any instant t is given by,

$$q = q_0\, e^{-t/RC}$$

$$\frac{q}{q_0} = e^{-t/RC}$$

$$\frac{q_0}{q} = e^{t/RC}$$

Taking log on both side

$$\log_e \frac{q_0}{q} = \frac{t}{RC}$$

$$R = \frac{t}{C \log_e \frac{q_0}{q}}$$

$$\boxed{R = \frac{t}{2.3026C \log_{10} \frac{q_0}{q}}} \qquad (1)$$

Using eq.(1), the value of R can determined on the knowledge of discharging time(t), capacitance (C) and q_0/q.

Method for determination of q_0/q:

For the determination0 of q_0/q *following circuit is designed (Fig.1.*

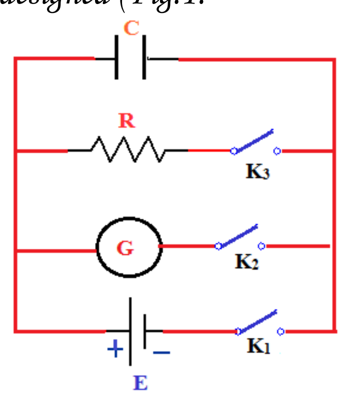

Fig.1

1. First of all K_1 *key is pressed for few minutes, so that capacitor is charged up to maximum value* q_0. *Now* K_1 *is released and*

K_2 is pressed. At this stage, the capacitor discharges through galvanometer which provides a deflection θ_0 corresponding to charge on capacitor q_0.

2. Now K_1 is again pressed after releasing K_2 so that capacitor is fully recharged. After it K3 is pressed for known time t. By this, charged capacitor discharges through resister. As a result, the instantaneous charge on it becomes q. This rest charge on capacitor is allowed to discharge with galvanometer by pressing key K_2. In this process, the galvanometer gives a deflection θ.

Since $q \propto \theta$

Thus, $q = k\,\theta \qquad (2)$

Similarly, $q_0 = k\,\theta_0 \qquad (3)$

Here k is a constant

From eqs.(2) and (3), we have,

$$\frac{q_0}{q} = \frac{\theta_0}{\theta} \qquad (4)$$

Substituting value from eq.(4) to eq.(1),

$$\boxed{R = \frac{t}{2.3026C\log_{10}\frac{\theta_0}{\theta}}} \qquad (5)$$

By the process (2), a series of t and q are obtained. A graph is then plotted $\log_{10}(\theta 0/\theta)$ verses t which comes a straight line (Fig.2). The slope of this line provides the mean value of $t/\log_{10}(\theta 0/\theta)$. Knowing the value of C, the value of R is calculated with of eq.(5).

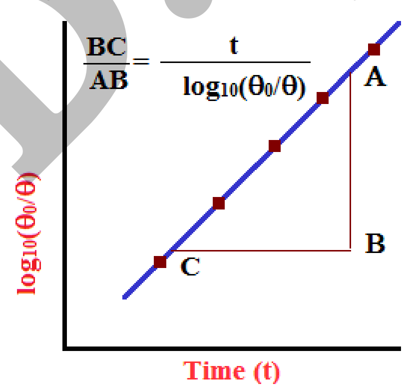

Fig.2

Note A: The small resistance is not be measured by this method because of low time constant for it. Due to low time constant, the capacitor discharges in very soon time. Thus, experimental measurement is not possible for this case.

Note B:

Unit of RC $= \text{ohm} \times \text{farad}$

$$= \frac{volt}{amp} \times \frac{coulomb}{volt}$$

$$= \frac{volt}{amp} \times \frac{amp \times \sec}{volt}$$

$$= \sec$$

Dimension of τ for RC circuit= dimension of time

Example 1: A capacitor is charged to a certain potential through a resistance of 30MΩ. If it reaches 3/4 of its final potential in 0.5sec then calculate the capacitance.

Solution: Given that,

R= 3MΩ, $V=3V_0/4$ and t=0.5 sec , C=?

We know that

$$V = V_0\left(1 - e^{-t/RC}\right)$$

$$\frac{3V_0}{4} = V_0\left(1 - e^{-t/RC}\right)$$

$$\frac{3}{4} = 1 - e^{-t/RC}$$

$$e^{-t/RC} = 1 - \frac{3}{4}$$

$$e^{-t/RC} = \frac{1}{4}$$

$$e^{t/RC} = 4 \quad \Rightarrow \frac{t}{RC} = \log_e 4$$

$$C = \frac{t}{R\log_e 4} = \frac{t}{2.3026R\log_{10} 4}$$

$$C = \frac{t}{2 \times 2.3026R\log_{10} 2}$$

$$C = \frac{0.5}{2 \times 2.3026 \times 3 \times 10^6 \times 0.3010}$$

$$C = \frac{0.5}{4.1585} \times 10^{-6} = 0.1202 \times 10^{-6} F$$

$$\boxed{C = 0.12\mu F}$$

Example 2: *A capacitor of capacitance 0.5µF is discharged through a resister. If half remains on capacitor in 3.47 sec then find the resistance?*

Solution: *Given that, C=0.5µF, t=3.45 sec, q=q₀/2, R=?*

$$R=\frac{t}{2.3026C\log_{10}\frac{q_0}{q}}$$

$$R=\frac{3.47}{2.3026\times0.5\times10^{-6}\times\log_{10}2}$$

$$R=\frac{3.47}{2.3026\times0.5\times10^{-6}\times0.3010}$$

$$R=\frac{3.47}{0.3465}\times10^{6}$$

$$R=\frac{3.47}{0.347}\times10^{6}=10\times10^{6}\Omega$$

$$\boxed{R=10M\Omega}$$

Example 3: *A capacitor of capacitance 4µF is charging with source of 12volts through a resister 1MΩ. Find out the instantaneous charge, voltage and current after time 4sec?*

Solution: *Given that, C=4µF , E=12volts, R=1MΩ*

q=?, V=? and i=?

$$\tau=RC=4\times10^{-6}\times1\times10^{6}=4\sec$$

(i)
$$q=q_0\left(1-e^{-t/\tau}\right)$$
$$q=CE\left(1-e^{-t/\tau}\right)$$
$$q=4\times10^{-6}\times12\times\left(1-e^{-4/4}\right)$$
$$q=48\times10^{-6}\times\left(1-e^{-1}\right)$$
$$q=48\times10^{-6}\times\left(1-\frac{1}{e}\right)$$
$$q=48\times10^{-6}\times\left(1-\frac{1}{2.718}\right)$$
$$q=48\times10^{-6}\times(1-0.368)$$
$$q=48\times10^{-6}\times0.632$$
$$q=30.34\times10^{-6}C$$
$$q=30.34\mu C$$

(ii)
$$V=q/C=\frac{30.34\times10^{-6}}{4\times10^{-6}}=7.585\text{ volt}$$

(iii)
$$I=I_0e^{-t/RC}=\frac{q_0}{\tau}e^{-t/\tau}$$
$$I=\frac{30.34\times10^{-6}}{1}e^{-1}=\frac{30.34\times10^{-6}}{e}$$
$$I=\frac{30.34\times10^{-6}}{2.718}=11.163\times10^{-6}amp$$
$$\boxed{I=11.1639\,\mu\text{A}}$$

Example4 *A capacitor of capacitance 5µF is discharged through a resister of resistance 10MΩ. Find the time in which charge on capacitor reduces to 36.8% of its maximum value.*

Solution: *Given that, R= 10MΩ, C=5µF ,*

If q=36.8%q₀, t=?

We know that when q=36.8%q₀, t=τ

$$t=\tau=RC=10\times10^{6}\times5\times10^{-6}$$

$$\boxed{t=50\ sec}$$

Example5 *When a charged capacitor of capacitance 1µF is connected through a galvanometer then it gives a deflection of 15cm. But it provides 10cm deflection when it discharged initially through a resister for 10sec. Find the value of resistance.*

Solution: *Given that, C=1µF, t=10 sec, θ₀=20cm, θ=15cm, R=?*

$$R=\frac{t}{2.3026C\log_{10}\frac{\theta_0}{\theta}}$$

$$R=\frac{10}{2.3026\times1\times10^{-6}\times\log_{10}(15/10)}$$

$$R=\frac{10\times10^{6}}{2.3026\times\log_{10}(3/2)}$$

$$R=\frac{10\times10^{6}}{2.3026\times(\log_{10}3-\log_{10}2)}$$

$$R=\frac{10\times10^{6}}{2.3026\times(0.4771-0.3010)}$$

$$R=\frac{10\times10^{6}}{2.3026\times0.1761}=\frac{10\times10^{6}}{0.4055}$$

$$\boxed{R=24.66M\Omega}$$