Lecture 1: Growth and decay of current in RL circuit

Growth of current in LR Circuit

Let us consider an inductor of self inductance L is connected to a DC source of e.m.f. E through a resister of resistance R and a key K in series. When the key K is switched on, the current in circuit started to increase. The current in the circuit does not attain the maximum steady state value (E/R) at once because the induced e.m.f. produced across the inductor $\left(-L\frac{dI}{dt}\right)$ *opposes the growth of current. Hence the current in the circuit increases slowly to attain its steady state value.*

According to KVL, the algebraic sum of instantaneous voltage drop across the circuit elements for a closed loop is zero. Thus, for the present RL circuit,

$$E - V_1 - V_2 = 0$$

$$E = V_1 + V_2 \qquad (1)$$

If at any instant the current in circuit is I then the potential drop across resister and inductor will be RI and $L\frac{dI}{dt}$ *respectively.*

Using eq.(1), we can write,

$$E = RI + L\frac{dI}{dt}$$

$$E - RI = L\frac{dI}{dt}$$

$$\frac{dI}{E - RI} = \frac{1}{L}dt \qquad (2)$$

Integrating eq.(2)

$$\int \frac{dI}{E - RI} = \frac{1}{L}\int dt + C'$$

$$-\frac{1}{R}\log_e(E - RI) = \frac{1}{L}t + C' \qquad (3)$$

Here C′ is integration constant and is determined by initial condition.

i.e. when t=0, I=0 then from eq.(3) we have

$$-\frac{1}{R}\log_e(E) = C' \qquad (4)$$

From eqs.(3) and (4), we can write,

$$-\frac{1}{R}\log_e(E - RI) = \frac{1}{L}t - \frac{1}{R}\log_e(E)$$

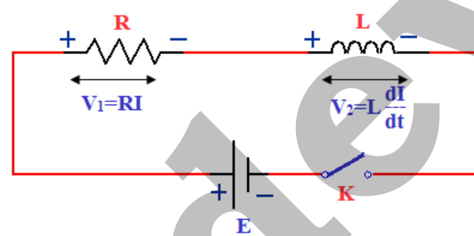

Fig 1

$$\log_e(E - RI) = -\frac{R}{L}t + \log_e(E)$$

$$\log_e(E - RI) - \log_e(E) = -\frac{R}{L}t$$

$$\log_e\frac{(E - RI)}{(E)} = -\frac{R}{L}t$$

$$\frac{E - RI}{E} = e^{-\frac{R}{L}t}$$

$$1 - \frac{R}{E}I = e^{-\frac{R}{L}t} \qquad \frac{R}{E}I = 1 - e^{-\frac{R}{L}t}$$

$$I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad (5)$$

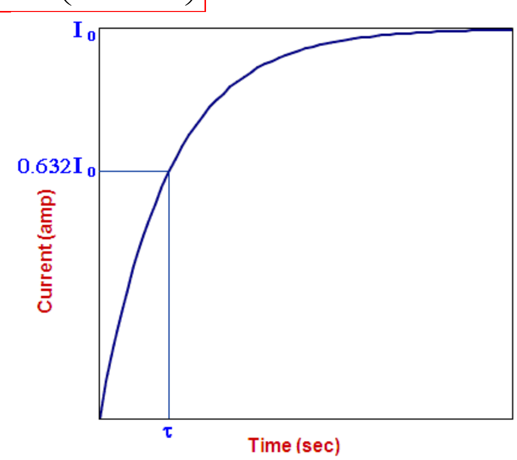

Fig.2

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Since , the steady state current or maximum current (I_0)= E/R

And dimension of L/R = dimension of time;

Say, $\frac{L}{R} = \tau =$ *inductive time constant*

Hence eq.(4) becomes as,

$$I = I_0\left(1 - e^{-\frac{t}{\tau}}\right) \qquad (6)$$

Eqs. (5) and (6) are called as expression of growth current in RL circuit.

These expressions indicates that-

(1) Initially the current in the circuit is zero.

(2) It increases exponentially following expression $(1 - e^{-t/\tau})$.

(3) After infinite time current reaches to its steady state value (I_0).

Inductive Time constant

The Growth of current in RL circuit follows following equation.

$$I = I_0\left\{1 - e^{-t/\tau}\right\}$$

When t=τ then from above eq., we can write

$$I = I_0\left\{1 - e^{-1}\right\}$$

$$I = I_0\{1 - (1/e)\}$$

$$I = I_0\{1 - (1/2.718)\}$$

$$I = I_0\{1 - 0.368\}$$

$$I = 0.632\, I_0$$

$$I = 0.632\, I_0 = 63.2\% I_0$$

Thus the time constant for the RL circuit is the time in which the current increases up to 63.2% of maximum current.

Decay of current in LR Circuit

Let us consider a charged inductor of self inductance L is connected to a resister of resistance R through a key K in series. When the key K is switched on, the inductor discharges through resister. The current in circuit started to decrease due to loss of inductive energy through resister.

According to KVL, the algebraic sum of instantaneous voltage drop across the circuit elements for a closed loop is zero.

Thus $E - V_1 - V_2 = 0$

$$E = V_1 + V_2$$

Here E=0, Thus $V_1 + V_2 = 0 \qquad (1)$

If at any instant the current in circuit is I then the potential drop across resister and inductor will be RI *and* $L\frac{dI}{dt}$ *respectively.*

Using eq.(1)

$$RI + L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} = -RI$$

$$\frac{dI}{I} = -\frac{R}{L}dt \qquad (2)$$

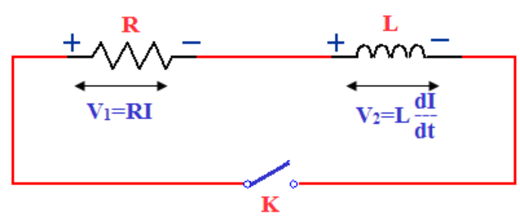

Fig.1

Integrating eq.(1)

$$\int\frac{dI}{I} = -\frac{R}{L}\int dt + C'$$

$$\log_e I = -\frac{R}{L}t + C' \qquad (3)$$

Here C' *is integration constant and is determined by initial condition.*

i.e. when t=0, I=I_0 then from eq.(3) we have

$$\log_e I_0 = C' \qquad (4)$$

From eqs. (3) and (4), we can write,

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$$\log_e I = -\frac{R}{L}t + \log_e I_0$$

$$\log_e \frac{I}{I_0} = -\frac{R}{L}t \qquad \Rightarrow \frac{I}{I_0} = e^{-\frac{R}{L}t}$$

$$I = I_0\, e^{-\frac{R}{L}t} \qquad (5)$$

Since , dimension of L/R = dimension of time;

Say, $\frac{L}{R} = \tau =$ *time constant*

Hence eq.(5) becomes as,

$$I = I_0\, e^{-t/\tau} \qquad (6)$$

Eqs. (5) and (6) are called as expression for decay of current in RL circuit. These expressions indicate that Initially the current has maximum value I_0 *and It decreases exponentially following expression*$(e^{-t/\tau})$.

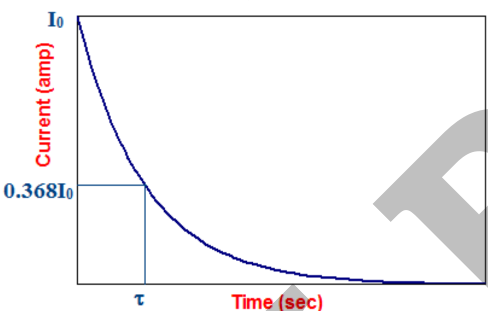


Fig.2

Inductive Time constant

The expression for decay of current in LR circuit is

$$I = I_0\, e^{-t/\tau}$$

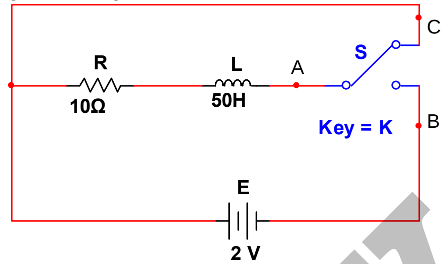
When t=τ then from above eq., we can write

$$I = I_0\, e^{-1} = I_0 \frac{1}{e} = I_0 \frac{1}{2.718} = 0.368\, I_0$$

$$I = 0.368\, I_0 = 36.8\% I_0$$

Thus the time constant for the RL circuit is the time in which the current decays from steady state value to 36.8% of maximum current.

Note A: *In the following LR circuit, when A is connected B then current rises in the circuit and inductor charges through resister. Furthermore, when A is connected to C then the charged inductor discharges through resister.*



In the Given circuit,

Maximum current=$I_0 = E/R = 2/10 = 0.2$ *amp*

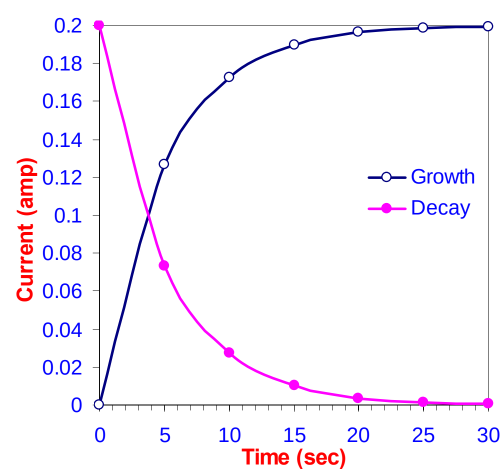
Time constant= $\tau = L/R = 50/10 = 5$ sec

The calculated growth and decay of current in the given circuit are presented in the Table A and is shown in Fig.A.

Table A

t (sec)	I_{growth} (amp)	I_{decay} (amp)
0	0	0.2
5	0.126424	0.073576
10	0.172933	0.027067
15	0.190043	0.009957
20	0.196337	0.003663
25	0.198652	0.001348
30	0.199504	0.000496

Fig. A



The above Table and Figure indicate that when t=5τ then the growth current increases up to approximately maximum value I_0 *and decay current decreases up to zero value.*

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Note B:

Unit of $\frac{L}{R} = \frac{henry}{ohm} = \frac{volt}{(amp/sec)\,ohm}$

$= \frac{volt\,.\sec}{amp\,.\,ohm} = \frac{volt\,.\sec}{volt} = \sec$

Dimension of τ for LR circuit= dimension of time

Note C: Rate of current growth in LR circuit

Since $I = I_0\left(1 - e^{-t/\tau}\right)$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{I_0}{\tau} e^{-\frac{t}{\tau}} \propto \frac{1}{\tau}$$

Hence the growth of current in LR circuit is The negative sign indicates that current decays with time.. i.e.

1. If τ: small then current growth: fast
2. If τ: large then current growth: slow

Note D: Rate of current decay in LR circuit

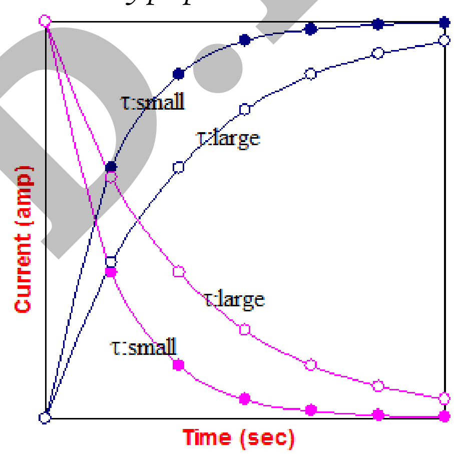
Since $I = I_0 e^{-\frac{t}{\tau}}$

$$\Rightarrow \quad \frac{dI}{dt} = -\frac{I_0}{\tau} e^{-\frac{t}{\tau}} \propto -\frac{1}{\tau}$$

Hence the growth of current in LR circuit is inversely proportional to time constant. The negative sign indicates that current decays with time..

1. If τ: small then current growth: fast
2. If τ: large then current growth: slow

Note E: Both growth and decay of current in LR circuit is inversely proportional to time constant.



Example 1: The time constant of an inductance coil is 2.5×10^{-3} sec. When 60 Ω resistance is added in series, the time constant reduces to 0.5×10^{-3} sec. Find the inductance and resistance of coil.

Solution: The time constant for LR circuit is τ and after addition of 60 Ω resistance, it becomes τ'.

$$\tau = \frac{L}{R} \qquad (1)$$

$$\tau' = \frac{L}{R+60} \qquad (2)$$

From eqs.(1) and (2), we have

$$\frac{\tau'}{\tau} = \frac{R}{R+60}$$

$$\frac{0.5\times10^{-3}}{2.5\times10^{-3}} = \frac{R}{R+60}$$

$$\frac{1}{5} = \frac{R}{R+60}$$

$$5R = R + 60$$

$$4R = 60$$

$$\boxed{R = 15\Omega}$$

Using eq.(1)

$$L = \tau R$$

$$L = 2.5\times10^{-3} \times 15$$

$$L = 37.5\times10^{-3}$$

$$\boxed{L = 3.75\times10^{-2}\,henry}$$

Example 2: An inductor of inductance 50henry and a resister of resistance 30Ω is connected to a d.c. source in series. Find the time in which the current reaches to half of maximum current in the circuit.

Solution: Given that,

L= 50 henry, R= 30 Ω, $I=I_0/2$ and t=?

$$I = I_0\left(1 - e^{-Rt/L}\right)$$

$$\frac{I_0}{2} = I_0\left(1 - e^{-30t/50}\right)$$

$$\frac{1}{2} = 1 - e^{-0.6t}$$

$$e^{-0.6t} = \frac{1}{2}$$

$$e^{0.6t} = 2$$

$$0.6t = \log_e 2$$

$$t = \frac{\log_e 2}{0.6} = \frac{2.3026 \log_{10} 2}{0.6}$$

$$t = \frac{2.3026 \times 0.3010}{0.6} = \frac{0.6931}{0.6}$$

$$\boxed{t = 1.155 \; sec}$$

Example 3: *An inductor of inductance 40henry and a resister of resistance 10Ω is connected to a d.c. source of 6volts. Find the current after 4 sec.*

Solution: *Given that,*

E=6 volts, L= 40 henry, R= 10 Ω, t=4sec, I=?

$$I = \frac{E}{R}\left(1 - e^{-Rt/L}\right) = \frac{6}{10}\left(1 - e^{-10\times 4/40}\right)$$

$$I = 0.6\left(1 - e^{-1}\right) = 0.6\left(1 - \frac{1}{e}\right)$$

$$I = 0.6 \times \left(1 - \frac{1}{2.713}\right) = 0.6 \times (1 - 0.368)$$

$$I = 0.6 \times 0.632$$

$$\boxed{I = 0.379 \; amp}$$

Example 4 *In an LR circuit with source, the current reaches to one third of its maximum value within 5sec. Find the time constant of the circuit.*

Solution: *Given that, $I=I_0/3$, τ=?*

$$I = I_0\left(1 - e^{-t/\tau}\right)$$

$$\frac{I_0}{3} = I_0\left(1 - e^{-t/\tau}\right)$$

$$\frac{1}{3} = \left(1 - e^{-t/\tau}\right)$$

$$e^{-t/\tau} = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow e^{t/\tau} = \frac{3}{2}$$

$$\frac{t}{\tau} = \log_e\left(\frac{3}{2}\right) = 2.3026 \times \log_{10}\left(\frac{3}{2}\right)$$

$$\frac{t}{\tau} = 2.3026 \times (\log_{10} 3 - \log_{10} 2)$$

$$\frac{t}{\tau} = 2.3026 \times (0.4771 - 0.3010)$$

$$\frac{t}{\tau} = 2.3026 \times (0.1761) = 0.4055$$

$$\tau = \frac{t}{0.4055} = \frac{5}{0.4055} = 12.33 \sec$$

$$\boxed{\tau = 12.33 \sec}$$

Example 5 *A resister of 5 Ω and an inductor of 4henry is connected to source of 10 volt in series. Find the time in which current in circuit becomes 1amp.*

Solution: *Given that, R= 5 Ω, L=4henry, E=10volts I=1amp, t=?*

$$I = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$$

$$1 = \frac{10}{5}\left(1 - e^{-5\times t/4}\right)$$

$$\frac{1}{2} = 1 - e^{-5\times t/4}$$

$$e^{-5\times t/4} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$e^{5\times t/4} = 2$$

$$\frac{5t}{4} = \log_e 2 = 2.3026 \log_{10} 2$$

$$t = \frac{4}{5} \times 2.3026 \times 0.3010 = 0.8 \times 0.6931$$

$$\boxed{t = 0.554 \sec}$$

Example 6 *A charged inductor of 40henry discharges through a resister of 5Ω. Find the time in which current decays to 36.8% of its maximum current.*

Solution: *Given that, R= 5Ω, L=40henry ,*
If I=36.8%I_0, t=?

We know that when I=36.8%I_0, t=τ

$$t = \tau = L/R = 40/5 = 8$$

$$\boxed{t = 8 \; sec}$$

Example 7 *The time constant for a RL circuit is 5sec. In case of decay of current, find the time in which current decays to half of its maximum.*

Solution: *Given that, τ= 5sec, $I=I_0/2$, t=?*

$$I = I_0 \, e^{-t/\tau}$$

$$\frac{I_0}{2} = I_0 \, e^{-t/5} \Rightarrow \frac{1}{2} = e^{-t/5}$$

$$e^{t/5} = 2$$

$$\frac{t}{5} = \log_e 2 = 2.3026 \log_{10} 2$$

$$t = 5 \times 2.3026 \times 0.3010 = 5 \times 0.6931$$

$$\boxed{t = 3.466 \sec}$$

