Electromagnetic Wave

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What happen ?

When the charge is static

Generated Field – Electric field

Method A

Point Charge

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Many Point Charges

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i^2}\hat{r}_i$$

Charges distribution

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}$$

$$dq = \lambda dl \;;\sigma ds \;;\rho dv$$

Method B

Gauss law

Net outward electric flux through closed surface =q/ε_0

$$\oint\vec{E}.d\vec{s} = q/\varepsilon_0$$

$$\oint\vec{E}.d\vec{s} = \frac{1}{\varepsilon_0}\oint\rho dv$$

$$\oint\left(\vec{\nabla}.\vec{E}\right)dv = \frac{1}{\varepsilon_0}\oint\rho dv$$

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

Method C

$$\vec{\nabla}\times\vec{E} = 0$$

$$\oint\vec{E}.d\vec{l} = 0$$

$$\vec{E} = -\vec{\nabla}\phi$$

$$\left|\vec{E}\right| = -\frac{d\phi}{dr}$$

Poisson's equation

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho d\tau}{r}$$

What happen ?

When the charge is moving with constant speed.

Generated Field – Magnetic field

Method A

Biot-Savart law

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\,\sin\theta}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\,d\vec{l} \times \vec{r}}{r^3}$$

Gauss law

$$\int \vec{B}.d\vec{s} = 0$$

$$\vec{\nabla} \bullet \vec{B} = 0$$

Method B

Ampere's Law

Line integral of mag. Field over a closed loop = $\mu_0 I$

$$\oint \vec{B}.d\vec{l} = \mu_0 I$$

$$\oint \vec{B}.d\vec{l} = \mu_0 \oint \vec{J}.d\vec{s}$$

$$\oint \left(\vec{\nabla} \times \vec{B}\right) d\vec{s} = \mu_0 \oint \vec{J}.d\vec{s}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Method C

$$\vec{\nabla} \times \vec{B} \neq 0$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

Poisson's equation

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}\,d\tau}{r}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I\,d\vec{l}}{r}$$

What happen ?

When the charge moves with an acceleration then

Field – Time varying electric and magnetic field

Time varying electric and magnetic field : ???
Variation of E & D and B & H with time : ???
Differential equation for E & D and B & H : ???
Magnitude of E & D and B & H : ???
Direction of E & D and B & H : ???

Maxwell Equation

Mathematical expressions based on the law's of electrostatics, magnetostatics and electromagnetic induction.

Useful in explanation of time varying electric and magnetic fields or EM wave .

First Maxwell Equation
Gauss Law of electrostatics

Net outward electric flux through closed surface =q/ε_0

$$d\phi = \vec{E} \bullet d\vec{s}$$
$$\phi = \oint \vec{E}.d\vec{s}$$

$$dq = \rho dv$$
$$q = \oint \rho dv$$

$$\oint \vec{E}.d\vec{s} = \frac{1}{\varepsilon_0} \oint \rho dv$$

Integral form of Ist Maxwell equation

$$\oint \left(\vec{\nabla}.\vec{E}\right) dv = \frac{1}{\varepsilon_0} \oint \rho dv$$

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$
$$\vec{\nabla}.\vec{D} = \rho$$

$$\vec{D} = \varepsilon_0 \vec{E}; \quad \vec{D} = \varepsilon \vec{E}$$

Differential form of Ist Maxwell equation

Second Maxwell Equation
Gauss Law of Magnetic field

Monopole can not exists.

Magnetic line of forces are closed curves.

Net outward magnetic =0

$$d\phi = \vec{B} \bullet d\vec{s}$$

$$\phi = \oint \vec{B}.d\vec{s}$$

$$\oint \vec{B}.d\vec{s} = 0$$

Integral form of IInd Maxwell equation

$$\oint \left(\vec{\nabla}.\vec{B}\right) dv = 0$$

$$\vec{\nabla}.\vec{B} = 0$$
$$\vec{\nabla}.\vec{H} = 0$$

Differential form of IInd Maxwell equation

$$\vec{B} = \mu_0 \vec{H}; \quad \vec{B} = \mu \vec{H}$$

Third Maxwell Equation

Faraday's Law of EM Induction

Induced emf = - rate of change of magnetic flux

$$e = -\frac{d\phi}{dt}$$

$$e = \oint \vec{E}.d\vec{l}$$

$$\phi = \oint \vec{B}.d\vec{s}$$

$$\oint \vec{E}.d\vec{l} = -\frac{d}{dt}\oint \vec{B}.d\vec{s}$$

Integral form of IIIrd Maxwell equation

$$\oint \left(\vec{\nabla} \times \vec{E}\right) d\vec{s} = -\oint \frac{d\vec{B}}{dt}.d\vec{s}$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$$

Differential form of IIIrd Maxwell equation

Fourth Maxwell Equation

Modified Ampere's Law by Maxwell

Line integral of magnetic field through closed path=μ_0I

$$\text{line integral of magnetic field} = \oint \vec{B}.d\vec{l}$$

$$dI = \vec{J}.d\vec{s}$$

$$\Rightarrow \quad \oint \vec{B}.d\vec{l} = \mu_0 \oint \vec{J}.d\vec{s}$$

$$\Rightarrow \quad \oint \left(\vec{\nabla} \times \vec{B}\right).d\vec{s} = \mu_0 \oint \vec{J}.d\vec{s}$$

$$\Rightarrow \quad \boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

This is Differential form of Ampere's Law.

Same result can be also obtained by Biot-Savart Law.

Fourth Maxwell Equation ....contd.

Modified Ampere's Law by Maxwell .....contd

Equation of continuity : Based on conservation of charge

$$\vec{\nabla}.\vec{J} + \frac{d\rho}{dt} = 0$$

J: Current density
ρ: Charge density

$$\because \quad \vec{\nabla}.\left(\vec{\nabla} \times \vec{B}\right) = \mu_0 \vec{\nabla}.\vec{J}$$

$$\therefore \quad \vec{\nabla}.\vec{J} = 0$$

$$\Rightarrow \quad \frac{d\rho}{dt} = 0$$

ρ = constant

So, Ampere's law and Biot savart Law are valid when charge density is constant or charges are moving with constant/uniform.

Fourth Maxwell Equation ….contd.

Modified Ampere's Law by Maxwell …..contd

New current: Due change in electric flux with time

New Current: Displacement Current

$$I_d = \varepsilon_0 \frac{d\phi_e}{dt} = \varepsilon_0 \frac{dEA}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

$$J_d = \frac{I_d}{A} = \varepsilon_0 \frac{dE}{dt} \Rightarrow \vec{J}_d = \varepsilon_0 \frac{d\vec{E}}{dt}$$

$$\vec{J}_d = \varepsilon_0 \frac{d\vec{E}}{dt} = \frac{d\vec{D}}{dt} \qquad \Rightarrow \vec{J}_{total} = \vec{J} + \vec{J}_d$$

$$\therefore \quad \vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \vec{J}_d \right)$$

$$\therefore \quad \vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_d$$

$$\oint \vec{B}.d\vec{l} = \mu_0 \oint \left(\vec{J} + \vec{J}_d \right) d\vec{s}$$

Integral form and Differential form of Fourth Maxwell equation

Maxwell Equation

Differential form of Maxwell equation

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon} \qquad \vec{\nabla}.\vec{D} = \rho$$

$$\vec{\nabla}.\vec{B} = 0 \qquad \vec{\nabla}.\vec{H} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \mu\left(\vec{J} + \vec{J}_d\right)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_d$$

Integral form of Maxwell equation

$$\oint \vec{E}.d\vec{s} = \frac{1}{\varepsilon}\oint \rho dv$$

$$\oint \vec{B}.d\vec{s} = 0$$

$$\oint \vec{E}.d\vec{l} = -\frac{d}{dt}\oint \vec{B}.d\vec{s}$$

$$\oint \vec{B}.d\vec{l} = \mu\oint\left(\vec{J} + \vec{J}_d\right).d\vec{s}$$

E and B in terms of ϕ and A

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} = -\frac{d(\vec{\nabla} \times \vec{A})}{dt} = -\vec{\nabla} \times \frac{d\vec{A}}{dt}$$

$$\vec{\nabla} \times \left(\vec{E} + \frac{d\vec{A}}{dt} \right) = 0 \quad \Rightarrow \quad \vec{E} + \frac{d\vec{A}}{dt} = -\vec{\nabla}\phi$$

$$\Rightarrow \quad \vec{E} = -\vec{\nabla}\phi - \frac{d\vec{A}}{dt}$$

Gauge Transformation: ϕ And A are taken in such a way that E and B remains invariant.

Lorentz Gauge Transformation:

$$\vec{\nabla}.\vec{A} + \mu_0 \varepsilon_0 \frac{d\phi}{dt} = 0$$

$$\nabla^2 \phi - \mu_0 \varepsilon_0 \frac{d^2\phi}{dt^2} = -\frac{\rho}{\varepsilon_0}$$

$$\nabla^2 \vec{A} - \mu_0 \varepsilon_0 \frac{d^2\vec{A}}{dt^2} = -\mu_0 \vec{J}$$

Medium

1. Isotropic Medium:

Microscopic properties (α, ε): direction independent

Velocity of em wave : direction independent

Example: air/free space/ vacuum, glass etc.

2. Anisotropic Medium

Microscopic properties (α, ε): direction dependent

Example: quartz, calcite etc.

Velocity of em wave : direction independent

3. Conducting Medium

Attenuating medium for em wave

Wave propagation vector: complex function

E and H: damped oscillation

Energy: decays with distance and time

Example: conductors.

Electromagnetic wave
In Free space

Properties of free space

$$\rho = 0;\ \ \sigma = 0;\ \ J = 0;\ \ \mu_r = 1;\ \ \varepsilon_r = 1;\ \ \mu = \mu_0;\ \ \varepsilon = \varepsilon_0$$

Maxwell Equation

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon}$$

$$\vec{\nabla}.\vec{H} = 0$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \varepsilon \frac{d\vec{E}}{dt}$$

Maxwell Equation for free space

$$\vec{\nabla}.\vec{E} = 0$$

$$\vec{\nabla}.\vec{H} = 0$$

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{d\vec{H}}{dt}$$

$$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{d\vec{E}}{dt}$$

Electromagnetic wave

In Free space: Differential Equation for E & H

Taking curl of IIIrd Maxwell Equation

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\mu_0 \frac{d}{dt}\left(\vec{\nabla}\times\vec{H}\right)$$

$$\vec{\nabla}\left(\vec{\nabla}.\vec{E}\right) - (\vec{\nabla}.\vec{\nabla})\vec{E} = -\mu_0 \frac{d}{dt}\left(\varepsilon_0 \frac{d\vec{E}}{dt}\right)$$

$$-\nabla^2\vec{E} = -\mu_0\varepsilon_0 \frac{d^2\vec{E}}{dt^2}$$

$$\nabla^2\vec{E} = \mu_0\varepsilon_0 \frac{d^2\vec{E}}{dt^2}$$

Taking curl of IV^th Maxwell Equation

$$\vec{\nabla}\times\vec{\nabla}\times\vec{H} = -\varepsilon_0 \frac{d}{dt}\left(\vec{\nabla}\times\vec{E}\right)$$

$$\vec{\nabla}\left(\vec{\nabla}.\vec{H}\right) - (\vec{\nabla}.\vec{\nabla})\vec{H} = -\varepsilon_0 \frac{d}{dt}\left(\mu_0 \frac{d\vec{H}}{dt}\right)$$

$$-\nabla^2\vec{H} = -\mu_0\varepsilon_0 \frac{d^2\vec{H}}{dt^2}$$

$$\nabla^2\vec{H} = \mu_0\varepsilon_0 \frac{d^2\vec{H}}{dt^2}$$

Progresive wave Equation

$$\nabla^2\psi = \frac{1}{v^2}\frac{d^2\psi}{dt^2}$$

E and H are progresive wave

$$\vec{E}(r,t) = \vec{E}_0\, e^{j\left(\vec{K}.\vec{r}-\omega t\right)}$$

$$\vec{H}(r,t) = \vec{H}_0\, e^{j\left(\vec{K}.\vec{r}-\omega t\right)}$$

Electromagnetic wave

In Free space: Nature of E and H

$\vec{\nabla}.\vec{E} = 0 \Rightarrow \vec{K}.\vec{E} = 0 \Rightarrow \vec{K} \perp \vec{E}$

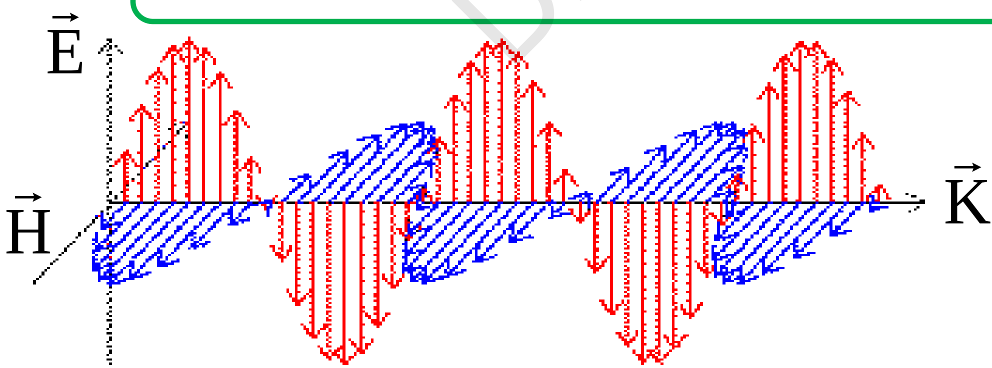
$\vec{\nabla}.\vec{H} = 0 \Rightarrow \vec{K}.\vec{H} = 0 \Rightarrow \vec{K} \perp \vec{H}$

$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{d\vec{H}}{dt} \Rightarrow \vec{K} \times \vec{E} = \omega\mu_0 \vec{H} \Rightarrow \vec{H} \perp \vec{K} \,\&\, \vec{E}$

$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{d\vec{E}}{dt} \Rightarrow \vec{K} \times \vec{H} = -\varepsilon_0 \omega \vec{E} \Rightarrow \vec{E} \perp \vec{K} \,\&\, \vec{H}$

$\Rightarrow$ $\vec{E}$, $\vec{H}$ and $\vec{K}$ are mutually perpendicular.

$\vec{K} = \frac{2\pi}{\lambda}\vec{n}$



$\vec{E}_X = \vec{E}_0\, e^{(k_z z - \omega t)}$

$\vec{H}_Y = \vec{H}_0\, e^{(k_z z - \omega t)}$

Electromagnetic wave

In Free space: velocity of em wave

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2}$$

$$\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{d^2 \vec{H}}{dt^2}$$

Comparing

$$\nabla^2 \psi = \frac{1}{v^2} \frac{d^2 \psi}{dt^2}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\frac{4\pi}{\mu_0} \times \frac{1}{4\pi\varepsilon_0}}$$

$$\Rightarrow \quad v = \sqrt{10^7 \times 9 \times 10^9} = 3 \times 10^8 = C$$

$$\Rightarrow \quad \boxed{v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = C}$$

Electromagnetic wave

Impedance of Free space for em wave

$$Z_0 = \text{Resistance offered by free space medium}$$

$$Z_0 = \frac{|\vec{E}|}{|\vec{H}|} = \frac{|\vec{E}_0|}{|\vec{H}_0|}$$

$$\because \quad \vec{K} \times \vec{E} = \omega\mu_0\vec{H}$$

$$\therefore \frac{E}{H} = \mu_0 \frac{\omega}{K} = \mu_0 v = \mu_0 c$$

$$Z_0 = \mu_0 c$$

$$Z_0 = \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \ \Omega \qquad \because v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

Electromagnetic wave

Energy density of EM wave in Free space

$$u = \text{Energy stored in unit volume}$$

$$u = u_e + u_m$$

$$\because \; u_e = \frac{1}{2}\varepsilon_0 E^2 \quad \& \quad u_m = \frac{1}{2}\mu_0 H^2$$

$$\therefore \; \frac{u_e}{u_m} = \frac{\varepsilon_0 E^2}{\mu_0 H^2} = \frac{\varepsilon_0 \mu_0}{\mu_0 \varepsilon_0} = 1 \quad \Rightarrow \boxed{u_e = u_m}$$

$$\boxed{u = u_e + u_e = 2u_e = \varepsilon_0 E^2}$$

$$\boxed{\langle u \rangle = \varepsilon_0 \langle E^2 \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \varepsilon_0 E_{rms}^2}$$

Electromagnetic wave

Poynting Vector in Free space

$\vec{S} =$ Energy flowing per unit area per unit time along $\vec{K}$

$\vec{S} =$ Power crossing per unit area along $\vec{K}$

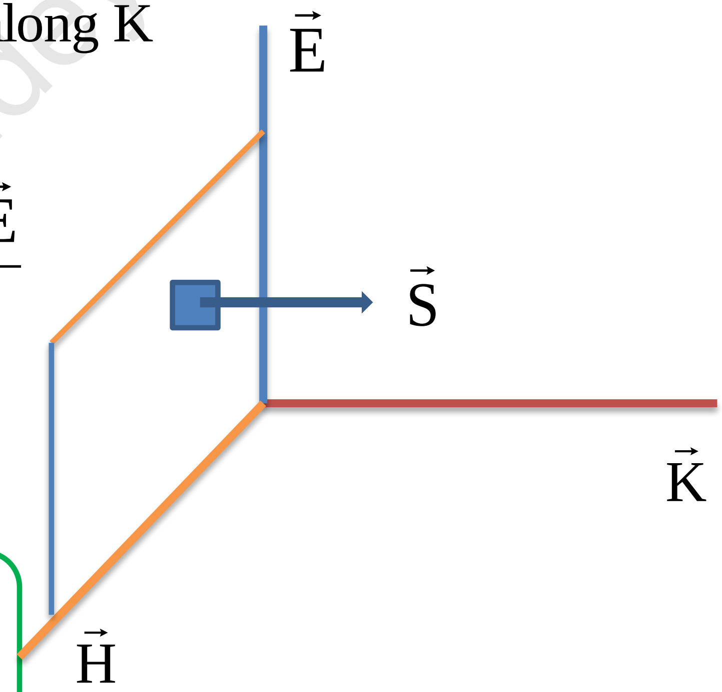
$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{S} = \vec{E} \times \frac{\vec{K} \times E}{\mu_0 \omega} = \vec{E} \times \frac{\vec{n} \times \vec{E}}{\mu_0 c}$$

$$\vec{S} = \frac{E^2}{\mu_0 c}\vec{n} = \frac{E^2}{Z_0}\vec{n}$$

$$\langle \vec{S} \rangle = \frac{E_{rms}^2}{\mu_0 c}\vec{n} = \frac{E_{rms}^2}{Z_0}\vec{n}$$

$$\langle \vec{S} \rangle = \langle u \rangle c\, \vec{n} \quad \Rightarrow \text{ energy flux} = \text{energy density} \times c$$



Electromagnetic wave

Poynting Theorem

$\vec{S}$ = Energy flowing per unit area per unit time along $\vec{K}$

$\vec{S}$ = Power crossing per unit area $\vec{K}$

$$-\vec{J}.\vec{E} = \frac{\partial u}{\partial t} + \vec{\nabla}.\vec{S}$$

- $-\vec{J}.\vec{E}$ = rate of energy transfered into em wave
  $-\vec{J}.\vec{E}$ = Power transfered into em field/wave
- $\frac{\partial u}{\partial t}$ = rate of change of electromagnetic energy
- $\vec{\nabla}.\vec{S}$ = energy flowing out through the boundary surface

EM wave in free space and isotropic mediums

Free Space	Isotropic medium

Properties of free space

Free Space	Isotropic medium
$\rho = 0; \ \sigma = 0; \ J = 0;$	$\rho = 0; \ \sigma = 0; \ J = 0;$
$\mu_r = 1; \ \varepsilon_r = 1;$	$\mu_r > 1; \ \varepsilon_r > 1;$
$\mu = \mu_0; \ \varepsilon = \varepsilon_0$	$\mu = \mu_r \mu_0; \ \varepsilon = \varepsilon_r \varepsilon_0$

Maxwell Equation

Free Space	Isotropic medium
$\vec{\nabla}.\vec{E} = 0$	$\vec{\nabla}.\vec{E} = 0$
$\vec{\nabla}.\vec{H} = 0$	$\vec{\nabla}.\vec{H} = 0$
$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{d\vec{H}}{dt}$	$\vec{\nabla} \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$
$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{d\vec{E}}{dt}$	$\vec{\nabla} \times \vec{H} = \varepsilon \frac{d\vec{E}}{dt}$

EM wave in free space and isotropic mediums

Free Space	*Isotropic medium*

Wave Equation

$$\nabla^2\vec{E} = \mu_0\varepsilon_0 \frac{d^2\vec{E}}{dt^2}$$

$$\nabla^2\vec{H} = \mu_0\varepsilon_0 \frac{d^2\vec{H}}{dt^2}$$

$$\nabla^2\vec{E} = \mu\varepsilon \frac{d^2\vec{E}}{dt^2}$$

$$\nabla^2\vec{H} = \mu\varepsilon \frac{d^2\vec{H}}{dt^2}$$

Wave Velocity

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = C$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{C}{\sqrt{\mu_r\varepsilon_r}} < C$$

Medium Impedance

$$Z_0 = \frac{E_0}{H_0} = \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ \Omega$$

$$Z = \frac{E}{H} = \mu_0 v = \sqrt{\frac{\mu}{\varepsilon}} = Z_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

EM wave in free space and isotropic mediums

Free Space	Isotropic medium

Transverse nature of EM wave

Free Space

$$\vec{K}.\vec{E}=0 \Longrightarrow \vec{K}\perp\vec{E}$$

$$\vec{K}.\vec{H}=0 \Longrightarrow \vec{K}\perp\vec{H}$$

$$\vec{K}\times\vec{E}=\omega\mu_0\vec{H} \Longrightarrow \vec{H}\perp\vec{K}\,\&\,\vec{E}$$

$$\vec{K}\times\vec{H}=-\varepsilon_0\omega\vec{E} \Longrightarrow \vec{E}\perp\vec{K}\,\&\,\vec{H}$$

Isotropic medium

$$\vec{K}.\vec{E}=0 \Longrightarrow \vec{K}\perp\vec{E}$$

$$\vec{K}.\vec{H}=0 \Longrightarrow \vec{K}\perp\vec{H}$$

$$\vec{K}\times\vec{E}=\omega\mu\vec{H} \Longrightarrow \vec{H}\perp\vec{K}\,\&\,\vec{E}$$

$$\vec{K}\times\vec{H}=-\varepsilon\omega\vec{E} \Longrightarrow \vec{E}\perp\vec{K}\,\&\,\vec{H}$$

Relation in E, B and K

Free Space

$$\vec{K}\times\vec{E}=\omega\mu_0\vec{H}$$

$$\vec{K}\times\vec{E}=\omega\vec{B}$$

$$\vec{B}=\frac{\vec{K}\times\vec{E}}{\omega}=\frac{\vec{n}\times\vec{E}}{v}=\frac{\vec{n}\times\vec{E}}{c}$$

Isotropic medium

$$\vec{K}\times\vec{E}=\omega\mu\vec{H}$$

$$\vec{K}\times\vec{E}=\omega\vec{B}$$

$$\vec{B}=\frac{\vec{K}\times\vec{E}}{\omega}=\frac{\vec{n}\times\vec{E}}{v}$$

EM wave in free space and isotropic mediums

Free Space	*Isotropic medium*

Energy density of EM wave

Free Space

$$u_e = \frac{1}{2}\varepsilon_0 E^2 \;\;\&\;\; u_m = \frac{1}{2}\mu_0 H^2$$

$$u_e = u_m \;\&\; u = 2u_e = \varepsilon_0 E^2$$

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \varepsilon_0 E_{rms}^2$$

Isotropic medium

$$u_e = \frac{1}{2}\varepsilon E^2 \;\;\&\;\; u_m = \frac{1}{2}\mu H^2$$

$$u_e = u_m \;\&\; u = 2u_e = \varepsilon E^2$$

$$\langle u \rangle = \frac{1}{2}\varepsilon E_0^2 = \varepsilon E_{rms}^2$$

Poynting Vector

Free Space

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{S} = \frac{E^2}{\mu_0 c}\vec{n} = \frac{E^2}{Z_0}\vec{n}$$

$$\langle \vec{S} \rangle = \frac{E_{rms}^2}{\mu_0 c}\vec{n} = \frac{E_{rms}^2}{Z_0}\vec{n}$$

Isotropic medium

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{S} = \frac{E^2}{\mu v}\vec{n} = \frac{E^2}{Z}\vec{n}$$

$$\langle \vec{S} \rangle = \frac{E_{rms}^2}{\mu v}\vec{n} = \frac{E_{rms}^2}{Z}\vec{n}$$

EM wave in conducting medium

Medium Properties

$$\rho \neq 0; \;\; \sigma \neq 0; \;\; J \neq 0; \mu_r > 1; \;\; \varepsilon_r > 1; \mu = \mu_r \mu_0; \;\; \varepsilon = \varepsilon_r \varepsilon_0$$

Maxwell Equation

$$\vec{\nabla}.\vec{E} = \rho / \varepsilon$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$$

$$\vec{\nabla}.\vec{H} = 0$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{d\vec{E}}{dt}$$

Wave Equation

$$\nabla^2 \vec{E} - \mu\sigma \frac{d\vec{E}}{dt} - \mu\varepsilon \frac{d^2\vec{E}}{dt^2} = 0$$

$$\nabla^2 \vec{H} - \mu\sigma \frac{d\vec{H}}{dt} - \mu\varepsilon \frac{d^2\vec{H}}{dt^2} = 0$$

EM wave in conducting medium ..contd

Wave propagation vector

$$\vec{K} = (\alpha + i\beta)\vec{n} = \vec{\alpha} + i\vec{\beta}$$

Solution of em wave Equation

$$\vec{E} = \vec{E}_0 e^{i(\vec{K}.\vec{r} - \omega t)} = \vec{E}_0\, e^{-\vec{\beta}.\vec{r}}\ e^{i(\vec{\alpha}.\vec{r} - \omega t)}$$

$$\vec{H} = \vec{H}_0 e^{i(\vec{K}.\vec{r} - \omega t)} = \vec{H}_0\, e^{-\vec{\beta}.\vec{r}}\ e^{i(\vec{\alpha}.\vec{r} - \omega t)}$$

Propagation and attenuation constants

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{(1+\tau^2} + 1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{(1+\tau^2} - 1\right]^{1/2}$$

$$\tau = \frac{\sigma}{\varepsilon\omega}$$

EM wave in conducting medium ..contd

Penetration depth or skin depth: **The distance at which amplitude of em wave becomes equal to 1/e times maximum value.**

$$E_0 e^{-\beta\delta} = E_0 e^{-1} \Rightarrow \delta = 1/\beta$$

For good Conductor

$$\because \tau = \frac{\sigma}{\omega\varepsilon} >> 1 \quad ; \quad \therefore \alpha = \beta = \sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

For Copper :

$$\sigma = 5.8\times10^{7}\ \text{mho/m};\ \ f = 1\text{MHz}$$

$$\delta = 6.6\times10^{-5}\,\text{m}$$

EM wave in conducting medium ..contd

Refractive Index of conducting medium

$$n^* = \frac{c}{v} = \frac{c}{\omega}K = \frac{c}{\omega}(\alpha + i\beta)$$

$$n^* = \frac{c}{\omega}\alpha + i\frac{c}{\omega}\beta = n + ik$$

Refractice Index

$$n = \frac{c}{\omega}\alpha$$

Attenuation constant

$$k = \frac{c}{\omega}\beta$$

For good Conductor

$$\because \tau = \frac{\sigma}{\omega\varepsilon} >> 1 \quad ; \quad \therefore \alpha = \beta = \sqrt{\frac{\mu\sigma\omega}{2}}$$

$$n = \sqrt{\frac{\mu\sigma}{2\omega}}$$

EM wave in conducting medium ..contd

Impedence of conducting medium

$$|Z| = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{(1+\tau^2)^{1/4}} \quad ; \; \tau = \frac{\sigma}{\omega\varepsilon}$$

E and H are out of Phase

$$\tan\phi = \tau = \frac{\sigma}{\omega\varepsilon}$$

Phase difference between E and H

$$\frac{\phi}{2} = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right) \approx \frac{\sigma}{2\omega\varepsilon}$$

For Perfect Conductor

$$\frac{\phi}{2} = \frac{\pi}{4}$$

EM wave in conducting medium ..contd

Energy density

$$u_e = \frac{1}{2}\varepsilon E_0^2\, e^{-2\vec{\beta}.\vec{r}} \cos^2(\vec{\alpha}.\vec{r} - \omega t)$$

$$\langle u_e \rangle = \frac{1}{4}\varepsilon E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

$$\langle u_m \rangle = \sqrt{1+\tau^2}\,\frac{1}{4}\varepsilon E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

$$\langle u_m \rangle = \sqrt{1+\tau^2}\,\langle u_e \rangle \qquad \langle u_m \rangle > \langle u_e \rangle$$

$$\langle u \rangle = \frac{1}{4}\left(1+\sqrt{1+\tau^2}\right)\varepsilon E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

For good Conductor

$$\langle u \rangle = \frac{1}{4}\frac{\sigma}{\omega} E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

Avg. Poynting Vector

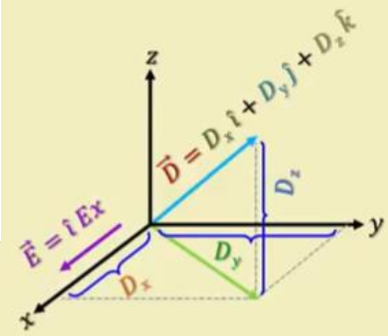
$$\langle S \rangle = \frac{1}{2}\sqrt{\frac{\sigma}{2\mu\omega}}\; E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

EM wave in Anisotropic medium

1. EM wave properties depends on direction

2. $J = 0; \rho = 0; \mu = \mu_0$

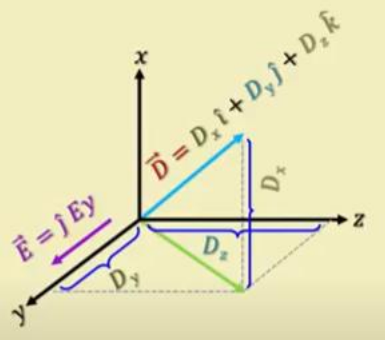
3. Permitivity is a tensor quantity.



$\vec{D}, \vec{E}$ Relationship in Anisotropic Media:

$\vec{E}$ field along arbitrary direction: $\vec{E} = \hat{\imath}\, E_x + \hat{\jmath}\, E_y + \hat{k}\, E_z$

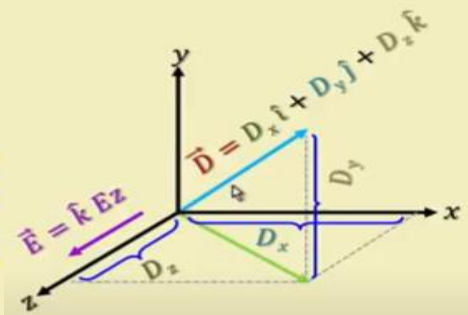
Then $\vec{D}$ field components are: $\vec{D} = \hat{\imath}\, D_x + \hat{\jmath}\, D_y + \hat{k}\, D_z$



$$D_x = \varepsilon_{xx} E_x + \varepsilon_{xy} E_y + \varepsilon_{xz} E_z$$

$$D_y = \varepsilon_{yx} E_x + \varepsilon_{yy} E_y + \varepsilon_{yz} E_z$$

$$D_z = \varepsilon_{zx} E_x + \varepsilon_{zy} E_y + \varepsilon_{zz} E_z$$



EM wave in Anisotropic medium

Permittivity tensor in Anisotropic Media:

In matrix notation:

$$\begin{pmatrix} D_x \\ D_y \\ D_x \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

i.e. $\vec{D} = \bar{\bar{\varepsilon}}\,\vec{E}$

$\bar{\bar{\varepsilon}}$ is a 3×3 permittivity tensor and symmetric in nature

$\varepsilon_{xy} = \varepsilon_{yx}$, $\varepsilon_{yz} = \varepsilon_{zy}$, $\varepsilon_{zx} = \varepsilon_{xz}$

EM wave in Anisotropic medium

Principal axes system:

- ✓ In general, the medium has <u>a set of orthogonal axes</u>
- ✓ Along these directions
  $\vec{D}$ field components <u>follow the direction of applied</u> $\vec{E}$
- ✓ This set of orthogonal axis

  ⇒ the Principal axis system

Permittivity tensor in Anisotropic Media:

In Principle axes system:

$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$$

ε_x, ε_y and ε_z are the principal dielectric permittivity components

So $\vec{D}, \vec{E}$ matrix equation:

$$\begin{pmatrix} D_x \\ D_y \\ D_x \end{pmatrix} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \Rightarrow \begin{matrix} D_x = \varepsilon_x E_x \\ D_y = \varepsilon_y E_y \\ D_z = \varepsilon_z E_z \end{matrix}$$

EM wave in Anisotropic medium

Permittivity property in general dielectric:

Anisotropic Medium: classification

$\varepsilon_x = \varepsilon_y = \varepsilon_z$ ⊂ Isotropic medium

$\varepsilon_x = \varepsilon_y \neq \varepsilon_z$ ⊂ Uniaxial medium

$\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$ ⊂ Biaxial medium

Refractive Indices in general dielectric:

$$n = \frac{V_1}{V_2} = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}} = \sqrt{\frac{\mu_0\varepsilon_2}{\mu_0\varepsilon_1}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

Refractive indices: $n_i = \sqrt{\frac{\varepsilon_i}{\varepsilon_0}}$ Along the principal axis

Isotropic medium: $n = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$ with $\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon$

Uniaxial medium:

ordinary R.I. $n_o = \sqrt{\frac{\varepsilon_x}{\varepsilon_0}} = \sqrt{\frac{\varepsilon_y}{\varepsilon_0}}$ extra-ordinary R.I. $n_e = \sqrt{\frac{\varepsilon_z}{\varepsilon_0}}$

EM wave in Anisotropic medium

Velocity of waves in Anisotropic Media:

velocity of waves ∝ RI of the medium $v = \frac{c}{n}$

- Isotropic medium ⊃ same along all directions
- Uniaxial medium ⊃ same along two directions
- Biaxial medium ⊃ different along all directions

✓ anisotropic – they can reorient the light
– contain one or two special directions called "optic axes" that do not reorient light

✓ media with one special direction are the uniaxial
✓ media with two special directions are the biaxial

EM wave in Anisotropic medium

Isotropic medium:

Velocity surface Spherical — Isotropic ⊃ Glass, Garnet

Anisotropic medium:

Velocity surface Ellipsoid — Uniaxial ⊃ Ice, Calcite, Quartz, Tourmaline

Biaxial ⊃ Mica, Topaz, Selenite

Positive and Negative uniaxial crystals

Quartz - Positive $(n_e - n_o) > 0$ $\quad n_e > n_o$

$n_o = 1.5443 \quad n_e = 1.5534$ $\quad v_e < v_o$

Calcite - Negative $(n_e - n_o) < 0$ $\quad n_e < n_o$

$n_o = 1.6584 \quad n_e = 1.4864$ $\quad v_e > v_o$

Velocity/RI is same along the OPTIC AXIS for ordinary and extraordinary wave

EM wave in Anisotropic medium

Plane waves in anisotropic medium

absence of free charge $\rho = 0$

absence of current ,i.e., $J = 0$

Maxwell's equations:

with $\vec{B}, \vec{H}$ relation : $\vec{B} = \mu_0 \vec{H}$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Fields of plane wave:

$$\vec{E} = \vec{E}_0 e^{i(\omega t - \vec{k}.\vec{r})} \qquad |\vec{H}| = \vec{H}_0 e^{i(\omega t - \vec{k}.\vec{r})}$$

$\vec{k}$ = wave vector of

$\omega = \frac{\omega}{c} n_\omega$ = frequency

n_ω = wave refractive indices

EM wave in Anisotropic medium

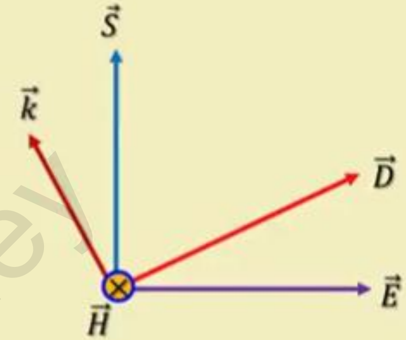
Relations and directions of $\vec{D}, \vec{H}, \vec{k}$

$$\vec{H} = \frac{(\vec{k} \times \vec{E})}{\omega\mu_0}$$

$\vec{H} \perp^r \vec{k}, \vec{E}$

$$\vec{D} = -\frac{(\vec{k} \times \vec{H})}{\omega}$$

$\vec{D} \perp^r \vec{k}, \vec{H}$



orthogonal triad of vectors

$\vec{k}, \vec{D}$ and $\vec{H}$ form a right handed Cartesian co-ordinate system

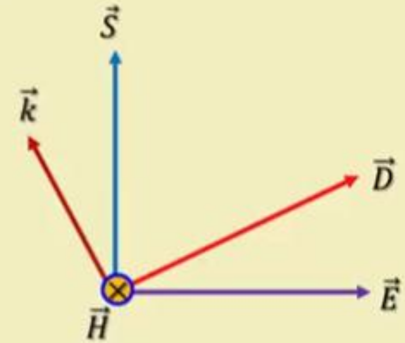
Relations and directions of $\vec{E}, \vec{H}, \vec{S}$

$$\vec{H} = \frac{(\vec{k} \times \vec{E})}{\omega\mu_0}$$

$\vec{H} \perp^r \vec{k}, \vec{E}$

$$\vec{S} = (\vec{E} \times \vec{H})$$

$\vec{S} \perp^r \vec{E}, \vec{H}$



orthogonal triad of vectors

$\vec{E}, \vec{H}$ and $\vec{S}$ form a right handed Cartesian co-ordinate system

Boundary condition for EM wave

1. **The normal component of electric displacement vector is not continuous at the interface but changes by an amount equal to the free surface charge density.**

$$D_{1n} - D_{2n} = \sigma$$

2. **The normal component of magnetic induction B is continuous across the interface.**

$$B_{1n} - B_{2n} = 0$$

Boundary condition for EM wave

3. **The tangential component of electric field is continuous at the interface.**

$$E_{1t} - E_{2t} = 0$$

4. **The tangential component of magnetic field strength is not continuous at the interface but changes by an amount equal to the component of the surface current density perpendicular to tangential component of H.**

$$H_{1t} - H_{2t} = J_{S\perp}$$

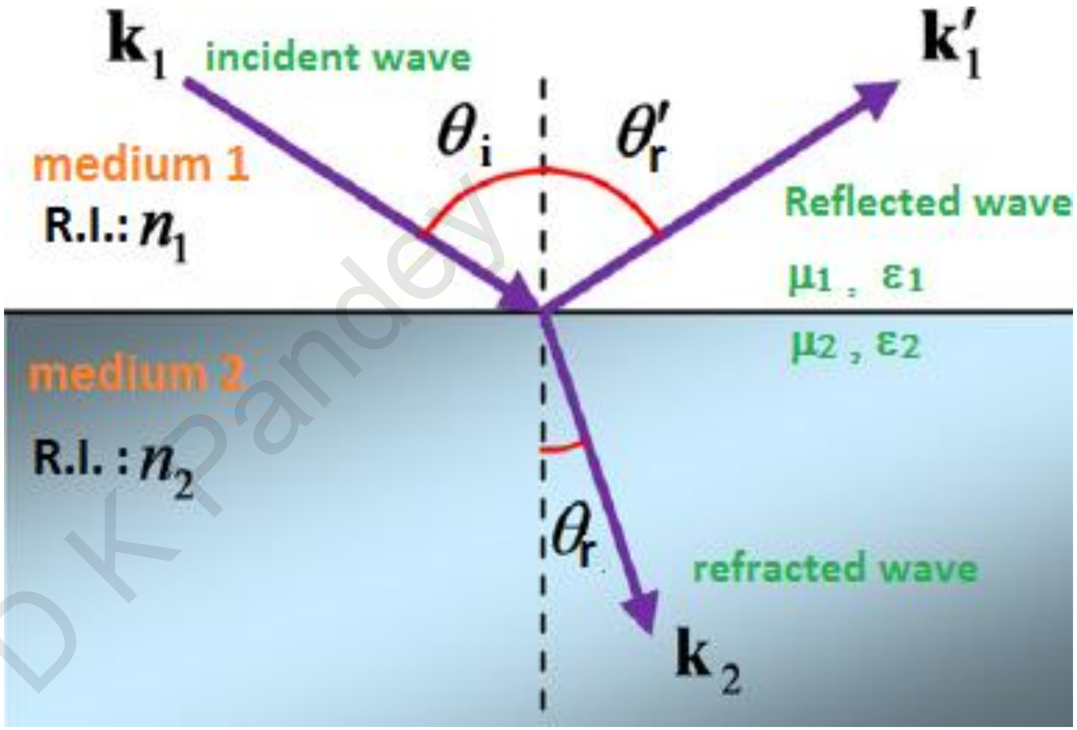
Reflection and Refraction of EM wave

For Incident wave

$$\vec{E}_1 = \vec{E}_{01} e^{i(\vec{K}_1.\vec{r}-\omega t)}$$

$$\vec{B}_1 = \frac{\vec{K}_1 \times \vec{E}_1}{\omega_1}$$

$$\vec{H}_1 = \frac{\vec{K}_1 \times \vec{E}_1}{\mu_1 \omega_1}$$



For reflected wave

$$\vec{E}_1' = \vec{E}_{01}' e^{i(\vec{K}_1'.\vec{r}-\omega t)}$$

$$\vec{B}_1' = \frac{\vec{K}_1' \times \vec{E}_1'}{\omega_1'}$$

$$\vec{H}_1' = \frac{\vec{K}_1' \times \vec{E}_1'}{\mu_1 \omega_1'}$$

For refracted wave

$$\vec{E}_2 = \vec{E}_{02} e^{i(\vec{K}_2.\vec{r}-\omega t)}$$

$$\vec{B}_2 = \frac{\vec{K}_2 \times \vec{E}_2}{\omega_2} \qquad \vec{H}_1 = \frac{\vec{K}_1 \times \vec{E}_1}{\mu_1 \omega_1}$$

Reflection and Refraction of EM wave

Since, tangential component of electric field is continuous at interface .

$$(E_1)_t + (E_1')_t = (E_2)_t$$

$$(E_{01})_t e^{i(\vec{k}_1.\vec{r}-\omega_1 t)} + (E_{01}')_t e^{i(\vec{k}_1'.\vec{r}-\omega_1' t)} = (E_{02})_t e^{i(\vec{k}_2.\vec{r}-\omega_2 t)}$$

$$\because \omega_1 = \omega_1' = \omega_2 = \omega$$

$$\therefore (E_{01})_t e^{i(\vec{k}_1.\vec{r})} + (E_{01}')_t e^{i(\vec{k}_1'.\vec{r})} = (E_{02})_t e^{i(\vec{k}_2.\vec{r})}$$

Therefore, for $(E_{01})_t + (E_{01}')_t = (E_{02})_t$

$$\therefore \quad (\vec{k}_1.\vec{r})_{z=0} = (\vec{k}_1'.\vec{r})_{z=0} = (\vec{k}_2.\vec{r})_{z=0}$$

Reflection and Refraction of EM wave

$$\text{If}, \quad (\vec{k}_1.\vec{r})_{z=0} = (\vec{k}_1'.\vec{r})_{z=0}$$

$$k_1 x \sin\theta_i = k_1' x \sin\theta_r'$$

Since wave propagation vector does not vary in same medium thus

$$\therefore k_1 = k_1'$$

$$\sin\theta_i = \sin\theta_r'$$

$$\theta_i = \theta_r'$$

Law of reflection

$$\text{If}, \quad (\vec{k}_1.\vec{r})_{z=0} = (\vec{k}_2.\vec{r})_{z=0}$$

$$k_1 x \sin\theta_i = k_2 x \sin\theta_r$$

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{k_2}{k_1}$$

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{k_2}{\omega}\frac{\omega}{k_1} = \frac{v_1}{v_2}$$

$$n = \frac{\sin\theta_i}{\sin\theta_r} = \frac{v_1}{v_2} = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}$$

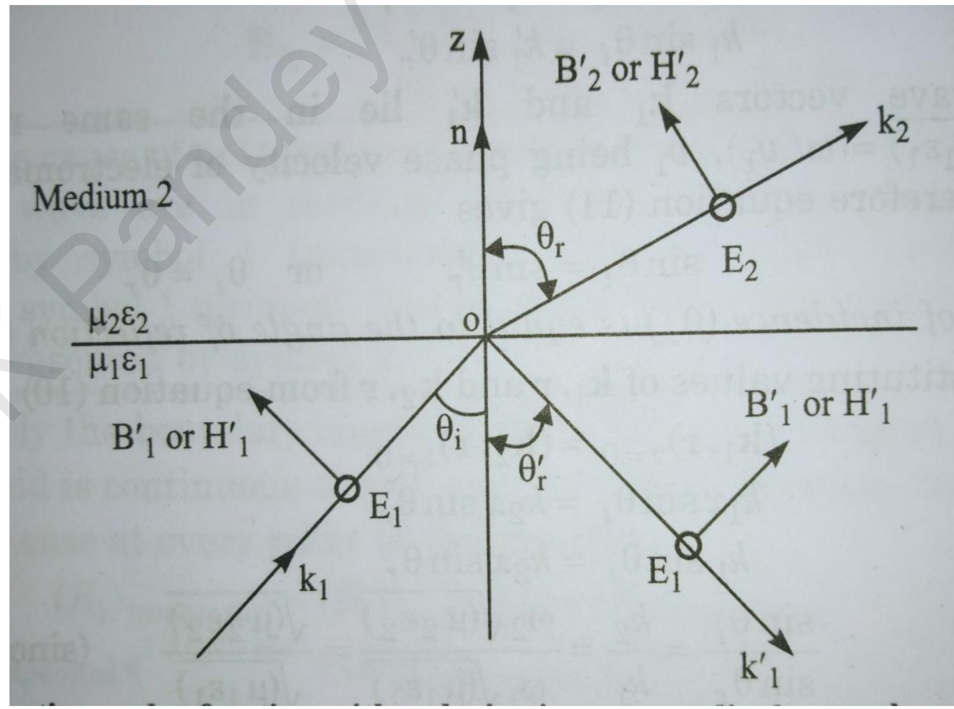
Law of refraction

Fresnel Equation for Reflection and Refraction of EM wave

A: When E is perpendicular to plane of incidence

$$\frac{E'_{01}}{E_{01}} = \frac{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_r}{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_r}$$

$$\frac{E_{02}}{E_{01}} = \frac{2\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i}{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_r}$$



Fresnel Equation………… contd for Reflection and Refraction of EM wave

$$\text{if} \quad \mu_1 = \mu_2 = \mu_0 \quad \text{then} \; \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\sin\theta_i}{\sin\theta_r}$$

$$\frac{E'_{01}}{E_{01}} = \frac{n_1\cos\theta_i - n_2\cos\theta_r}{n_1\cos\theta_i + n_2\cos\theta_r} = -\frac{\sin(\theta_i - \theta_r)}{\sin(\theta_i + \theta_r)}$$

$$\frac{E_{02}}{E_{01}} = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_r} = \frac{2\cos\theta_i\sin\theta_r}{\sin(\theta_i + \theta_r)}$$

For normal incidence ; $\theta_i = \theta_r = 0$

$$\frac{E'_{01}}{E_{01}} = \frac{n_1 - n_2}{n_1 + n_2} \qquad \frac{E_{02}}{E_{01}} = \frac{2n_1}{n_1 + n_2}$$

Fresnel Equation………… contd for Reflection and Refraction of EM wave

Case 1: when $(n_1/n_2)<1$; wave : rare to denser

$n = \sin\theta_i / \sin\theta_r = (n_2/n_1) > 1$ and $\theta_i > \theta_r$

$$\frac{E'_{01}}{E_{01}} = -ve \Rightarrow \text{Phase Change} = \pi \text{ ; Path difference} = \frac{\lambda}{2}$$

$$\frac{E_{02}}{E_{01}} = +ve \Rightarrow \text{Phase Change} = 0$$

Reflected and incident wave are in opposite phase.

Refracted and incident wave are in same phase.

Case 2: when $(n_1/n_2)>1$; wave :denser to rare

$n = \sin\theta_i / \sin\theta_r = (n_2/n_1) < 1$ and $\theta_i < \theta_r$

$$\frac{E'_{01}}{E_{01}} = +ve \Rightarrow \text{Phase Change} = 0$$

$$\frac{E_{02}}{E_{01}} = +ve \Rightarrow \text{Phase Change} = 0$$

Reflected and incident wave are in same phase.

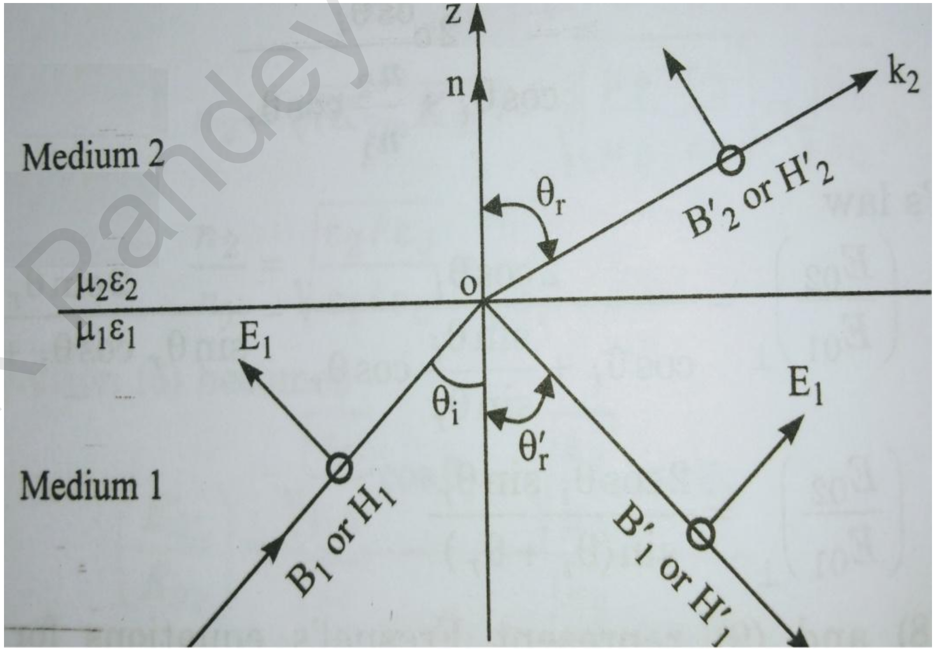
Refracted and incident wave are in same phase.

Fresnel Equation………… contd
for Reflection and Refraction of EM wave

B: When E is parallel to plane of incidence

$$\frac{E'_{01}}{E_{01}} = \frac{\sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_i - \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_r}{\sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_i + \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_r}$$

$$\frac{E_{02}}{E_{01}} = \frac{2\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_i}{\sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_i + \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_r}$$



Fresnel Equation………… contd for Reflection and Refraction of EM wave

$$\text{if}\quad \mu_1 = \mu_2 = \mu_0 \quad \text{then}\ \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\sin\theta_i}{\sin\theta_r}$$

$$\frac{E'_{01}}{E_{01}} = \frac{n_2\cos\theta_i - n_1\cos\theta_r}{n_2\cos\theta_i + n_1\cos\theta_r} = \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)}$$

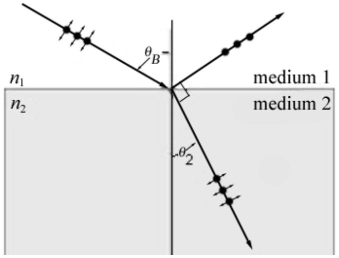
$$\frac{E_{02}}{E_{01}} = \frac{2n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_r} = \frac{2\cos\theta_i\sin\theta_r}{\sin(\theta_i + \theta_r)\cos(\theta_i - \theta_r)}$$

For normal incidence ; $\theta_i=\theta_r=0$

$$\frac{E'_{01}}{E_{01}} = \frac{n_2 - n_1}{n_2 + n_1} \qquad \frac{E_{02}}{E_{01}} = \frac{2n_1}{n_2 + n_1}$$

Fresnel Equation………… Contd….
for Reflection and Refraction of EM wave

Case 1: $$\frac{E_{02}}{E_{01}} = +ve \Rightarrow \text{Phase Change} = 0$$



Refracted and incident wave are in same phase.

Case 2: From both cases we found that -

$$r_{II} = \left(\frac{E'_{01}}{E_{01}}\right)_{II} = \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)} \quad ; \quad r_{\perp} = \left(\frac{E'_{01}}{E_{01}}\right)_{\perp} = -\frac{\sin(\theta_i - \theta_r)}{\sin(\theta_i + \theta_r)}$$

$$\text{if} \quad \theta_i + \theta_r = \frac{\pi}{2} \quad \text{or} \quad \theta_p = \theta_B = \theta_i = \frac{\pi}{2} - \theta_r = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

$$r_{II} = 0 \quad \text{and} \quad r_{\perp} \neq 0$$

Thus , if an un-polarized light incidents (rare to denser) at angle **$\theta p = \theta_B$** then only electric vector perpendicular to plane of incidence will be reflected and light will become polarized. This angle is termed as angle of polarization or Brewster's angle.

Fresnel Equation………… Contd….
for Reflection and Refraction of EM wave

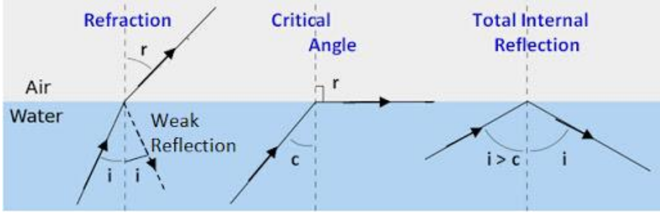
Case 3: if em wave travels from denser to rarer medium then it goes away from normal.

$$\theta_r = \frac{\pi}{2} \quad \text{or} \quad \theta_i = \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$r_{II} = \left(\frac{E'_{01}}{E_{01}}\right)_{II} = \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)} \quad ; \quad r_{\perp} = \left(\frac{E'_{01}}{E_{01}}\right)_{\perp} = -\frac{\sin(\theta_i - \theta_r)}{\sin(\theta_i + \theta_r)}$$

$$r_{II} = 1 \ \text{ and } \ r_{\perp} = 1$$

$$R_{II} = (r_{II})^2 = 1 \ \text{ and } \ R_{\perp} = (r_{\perp})^2 = 1$$



Thus total energy is reflected at the interface of two mediums.

This means, as the angle of incident wave increases, the intensity of reflected wave increases while intensity of refracted wave diminishes. The intensity of refracted wave becomes zero at $\theta_i = \theta_c$. But if $\theta i > \theta c$ then wave completely goes in medium Ist, and follows law of reflection. This is TIR.

A Lot of Thanks

for kind attention