EM Wave in Different Medium

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Medium

1. Isotropic Medium:

Microscopic properties (α, ε): direction independent

Velocity of em wave : direction independent

Example: air/free space/ vacuum, glass etc.

2. Anisotropic Medium

Microscopic properties (α, ε): direction independent

Example: quartz, calcite etc.

Velocity of em wave : direction independent

3. Conducting Medium

Attenuating medium for em wave

Wave propagation vector: complex function

E and H: damped oscillation

Energy: decays with distance and time

Example: conductors.

EM wave in free space and isotropic mediums

Free Space	*Isotropic medium*

Properties of free space

Free Space	Isotropic medium
$\rho = 0;\ \sigma = 0;\ J = 0;$	$\rho = 0;\ \sigma = 0;\ J = 0;$
$\mu_r = 1;\ \varepsilon_r = 1;$	$\mu_r > 1;\ \varepsilon_r > 1;$
$\mu = \mu_0;\ \varepsilon = \varepsilon_0$	$\mu = \mu_r\mu_0;\ \varepsilon = \varepsilon_r\varepsilon_0$

Maxwell Equation

Free Space	Isotropic medium
$\vec{\nabla}.\vec{E} = 0$	$\vec{\nabla}.\vec{E} = 0$
$\vec{\nabla}.\vec{H} = 0$	$\vec{\nabla}.\vec{H} = 0$
$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{d\vec{H}}{dt}$	$\vec{\nabla} \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$
$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{d\vec{E}}{dt}$	$\vec{\nabla} \times \vec{H} = \varepsilon \frac{d\vec{E}}{dt}$

EM wave in free space and isotropic mediums

Free Space	Isotropic medium

Wave Equation

$$\nabla^2\vec{E} = \mu_0\varepsilon_0 \frac{d^2\vec{E}}{dt^2}$$

$$\nabla^2\vec{H} = \mu_0\varepsilon_0 \frac{d^2\vec{H}}{dt^2}$$

$$\nabla^2\vec{E} = \mu\varepsilon \frac{d^2\vec{E}}{dt^2}$$

$$\nabla^2\vec{H} = \mu\varepsilon \frac{d^2\vec{H}}{dt^2}$$

Wave Velocity

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = C$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{C}{\sqrt{\mu_r \varepsilon r}} < C$$

Medium Impedance

$$Z_0 = \frac{E_0}{H_0} = \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ \Omega$$

$$Z = \frac{E}{H} = \mu_0 v = \sqrt{\frac{\mu}{\varepsilon}} = Z_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

EM wave in free space and isotropic mediums

Free Space	Isotropic medium

Transverse nature of EM wave

Free Space

$\vec{K}.\vec{E} = 0 \Rightarrow \vec{K} \perp \vec{E}$

$\vec{K}.\vec{H} = 0 \Rightarrow \vec{K} \perp \vec{H}$

$\vec{K} \times \vec{E} = \omega\mu_0\vec{H} \Rightarrow \vec{H} \perp \vec{K} \,\&\, \vec{E}$

$\vec{K} \times \vec{H} = -\varepsilon_0\omega\vec{E} \Rightarrow \vec{E} \perp \vec{K} \,\&\, \vec{H}$

Isotropic medium

$\vec{K}.\vec{E} = 0 \Rightarrow \vec{K} \perp \vec{E}$

$\vec{K}.\vec{H} = 0 \Rightarrow \vec{K} \perp \vec{H}$

$\vec{K} \times \vec{E} = \omega\mu\vec{H} \Rightarrow \vec{H} \perp \vec{K} \,\&\, \vec{E}$

$\vec{K} \times \vec{H} = -\varepsilon\omega\vec{E} \Rightarrow \vec{E} \perp \vec{K} \,\&\, \vec{H}$

Relation in E, B and K

Free Space

$$\vec{K} \times \vec{E} = \omega\mu_0\vec{H}$$

$$\vec{K} \times \vec{E} = \omega\vec{B}$$

$$\vec{B} = \frac{\vec{K} \times \vec{E}}{\omega} = \frac{\vec{n} \times \vec{E}}{v} = \frac{\vec{n} \times \vec{E}}{c}$$

Isotropic medium

$$\vec{K} \times \vec{E} = \omega\mu\vec{H}$$

$$\vec{K} \times \vec{E} = \omega\vec{B}$$

$$\vec{B} = \frac{\vec{K} \times \vec{E}}{\omega} = \frac{\vec{n} \times \vec{E}}{v}$$

EM wave in free space and isotropic mediums

Free Space	Isotropic medium

Energy density of EM wave

Free Space

$$u_e = \frac{1}{2}\varepsilon_0 E^2 \ \ \& \ \ u_m = \frac{1}{2}\mu_0 H^2$$

$$u_e = u_m \ \& \ u = 2u_e = \varepsilon_0 E^2$$

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \varepsilon_0 E_{rms}^2$$

Isotropic medium

$$u_e = \frac{1}{2}\varepsilon E^2 \ \ \& \ \ u_m = \frac{1}{2}\mu H^2$$

$$u_e = u_m \ \& \ u = 2u_e = \varepsilon E^2$$

$$\langle u \rangle = \frac{1}{2}\varepsilon E_0^2 = \varepsilon E_{rms}^2$$

Poynting Vector

Free Space

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{S} = \frac{E^2}{\mu_0 c}\vec{n} = \frac{E^2}{Z_0}\vec{n}$$

$$\langle \vec{S} \rangle = \frac{E_{rms}^2}{\mu_0 c}\vec{n} = \frac{E_{rms}^2}{Z_0}\vec{n}$$

$$\langle \vec{S} \rangle = \langle u \rangle c\, \vec{n}$$

Isotropic medium

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{S} = \frac{E^2}{\mu v}\vec{n} = \frac{E^2}{Z}\vec{n}$$

$$\langle \vec{S} \rangle = \frac{E_{rms}^2}{\mu v}\vec{n} = \frac{E_{rms}^2}{Z}\vec{n}$$

$$\langle \vec{S} \rangle = \langle u \rangle v\, \vec{n}$$

EM wave in conducting medium

Medium Properties

$$\rho \neq 0; \; \sigma \neq 0; \; J \neq 0; \mu_r > 1; \; \varepsilon_r > 1; \mu = \mu_r \mu_0; \; \varepsilon = \varepsilon_r \varepsilon_0$$

Maxwell Equation

$$\vec{\nabla} . \vec{E} = \rho / \varepsilon$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$$

$$\vec{\nabla} . \vec{H} = 0$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{d\vec{E}}{dt}$$

Wave Equation

$$\nabla^2 \vec{E} - \mu\sigma \frac{d\vec{E}}{dt} - \mu\varepsilon \frac{d^2\vec{E}}{dt^2} = 0$$

$$\nabla^2 \vec{H} - \mu\sigma \frac{d\vec{H}}{dt} - \mu\varepsilon \frac{d^2\vec{H}}{dt^2} = 0$$

EM wave in conducting medium ..contd

Wave propagation vector

$$\vec{K} = (\alpha + i\beta)\vec{n} = \vec{\alpha} + i\vec{\beta}$$

Solution of em wave Equation

$$\vec{E} = \vec{E}_0 e^{i(\vec{K}.\vec{r} - \omega t)} = \vec{E}_0\, e^{-\vec{\beta}.\vec{r}}\; e^{i(\vec{\alpha}.\vec{r} - \omega t)}$$

$$\vec{H} = \vec{H}_0 e^{i(\vec{K}.\vec{r} - \omega t)} = \vec{H}_0\, e^{-\vec{\beta}.\vec{r}}\; e^{i(\vec{\alpha}.\vec{r} - \omega t)}$$

Propagation and attenuation constants

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{(1+\tau^2} + 1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{(1+\tau^2} - 1\right]^{1/2}$$

$$\tau = \frac{\sigma}{\varepsilon\omega}$$

EM wave in conducting medium ..contd

Velocity of EM Wave

$$v = \frac{\omega}{\alpha} = \sqrt{\frac{2}{\mu\varepsilon}}\left[\sqrt{1+\tau^2}+1\right]^{-1/2} ; \tau = \frac{\sigma}{\omega\varepsilon}$$

Criterion for a medium to be Conductor or dielectric

$$\frac{\vec{J}_{cond.}}{\vec{J}_D} = \frac{\sigma\vec{E}}{\varepsilon\frac{d\vec{E}}{dt}} = i\frac{\sigma}{\omega\varepsilon} = i\tau$$

For good Conductor

$$\tau = \frac{\sigma}{\omega\varepsilon} >> 1$$

For good Dielectric

$$\tau = \frac{\sigma}{\omega\varepsilon} << 1$$

For copper $\tau = \frac{10^{18}}{\omega}\sec$

For insulator $\tau = \frac{10^{-4}}{\omega}\sec$

EM wave in conducting medium ..contd

Penetration depth or skin depth: **The distance at which amplitude of em wave becomes equal to 1/e times maximum value.**

$$E_0 e^{-\beta\delta} = E_0 e^{-1} \Rightarrow \boxed{\delta = 1/\beta}$$

For good Conductor

$$\because \tau = \frac{\sigma}{\omega\varepsilon} >> 1 \quad ; \quad \therefore \alpha = \beta = \sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\boxed{\delta = \sqrt{\frac{2}{\mu\sigma\omega}}}$$

For Copper :

$$\sigma = 5.8\times10^{7}\ \text{mho/m};\ \ f = 1\text{MHz}$$

$$\delta = 6.6\times10^{-5}\,\text{m}$$

EM wave in conducting medium ..contd

Refractive Index of conducting medium

$$n^* = \frac{c}{v} = \frac{c}{\omega}K = \frac{c}{\omega}(\alpha + i\beta)$$

$$n^* = \frac{c}{\omega}\alpha + i\frac{c}{\omega}\beta = n + ik$$

Refractice Index

$$n = \frac{c}{\omega}\alpha$$

Attenuation constant

$$k = \frac{c}{\omega}\beta$$

For good Conductor

$$\because \tau = \frac{\sigma}{\omega\varepsilon} >> 1 \quad ; \quad \therefore \alpha = \beta = \sqrt{\frac{\mu\sigma\omega}{2}}$$

$$n = \sqrt{\frac{\mu\sigma}{2\omega}}$$

EM wave in conducting medium ..contd

Impedence of conducting medium

$$|Z| = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{(1+\tau^2)^{1/4}} \quad ; \ \tau = \frac{\sigma}{\omega\varepsilon}$$

E and H are out of Phase

$$\tan\phi = \tau = \frac{\sigma}{\omega\varepsilon}$$

Phase difference between E and H

$$\frac{\phi}{2} = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right) \approx \frac{\sigma}{2\omega\varepsilon}$$

For Perfect Conductor

$$\frac{\phi}{2} = \frac{\pi}{4}$$

EM wave in conducting medium ..contd

Energy density

$$u_e = \frac{1}{2}\varepsilon E_0^2\, e^{-2\vec{\beta}.\vec{r}} \cos^2(\vec{\alpha}.\vec{r} - \omega t)$$

$$\langle u_e \rangle = \frac{1}{4}\varepsilon E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

$$\langle u_m \rangle = \sqrt{1+\tau^2}\,\frac{1}{4}\varepsilon E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

$$\langle u_m \rangle = \sqrt{1+\tau^2}\,\langle u_e \rangle \qquad \langle u_m \rangle > \langle u_e \rangle$$

$$\langle u \rangle = \frac{1}{4}\left(1+\sqrt{1+\tau^2}\right)\varepsilon E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

For good Conductor

$$\langle u \rangle = \frac{1}{4}\frac{\sigma}{\omega} E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

Avg. Poynting Vector

$$\langle S \rangle = \frac{1}{2}\sqrt{\frac{\sigma}{2\mu\omega}}\; E_0^2\, e^{-2\vec{\beta}.\vec{r}}$$

Boundary condition for EM wave

1. The normal component of electric displacement vector is not continuous at the interface but changes by an amount equal to the free surface charge density.

$$D_{1n} - D_{2n} = \sigma$$

2. The normal component of magnetic induction B is continuous across the interface.

$$B_{1n} - B_{2n} = 0$$

Boundary condition for EM wave

3. The tangential component of electric field is continuous at the interface.

$$E_{1t} - E_{2t} = 0$$

4. The tangential component of magnetic field strength is not continuous at the interface but changes by an amount equal to the component of the surface current density perpendicular to tangential component of H.

$$H_{1t} - H_{2t} = J_{S\perp}$$

A Lot of Thanks

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